Instructions:

1. Supply the information requested at the top of each page.
2. Show your work in the space provided. If you need more space, use the back of the previous page or the blank page at the end of the test, and indicate (in the space provided) where your work is continued.
3. Highest marks are given for work that is well reasoned, complete, and presented in a clear and concise way.
4. This test has 8 questions of equal weight on 10 pages.

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1. Algebra and Geometry

Let P:**p**, Q:**q**, and R:**r** be three (distinct) points in the projective plane. Prove that P, Q, and R are collinear if and only if there are three real, non-zero numbers a, b, and c so that a**p**+b**q**+c**r**=**0**.

2. Theorem of Pappus

State and prove the Theorem of Pappus.

3. Collineations

Find the matrix of the collineation that maps the standard frame of reference to the frame of reference given by {(1,3,0), (3,0,0), (1,1,5), (3,2,4)}.

4. Conic in the projective plane

The Cartesian description of a conic is given as follows:

(a) It is asymptotic to the x-axis.
(b) It is asymptotic to the y-axis.
(c) It contains the point (3,2).

4.1 Translate these three statements to the corresponding algebraic conditions for the equation of the conic in the real projective plane.

4.2 Use the matrix representation for a non-degenerate conic in the real projective plane to find the homogeneous equation for this conic.

4.3 Write the same equation in Cartesian form. (6xy=1).

5. Inversion

A point P and a circle C are given with P distinct from the centre of C. Give two constructions for the inverse of P with respect to C. Each construction is to be a "straightedge and compass" construction. (You are not asked to prove that your constructions are correct.)

6. Stereographic projection

πThe point N:(0,0,1) is on the sphere with centre (0,0,0) and radius 1. Consider the stereographic projection through N from the points on the sphere to the points of the xy-plane.

6.1 Derive the equation for this stereographic projection.

6.2 Find the equation of the plane in three space that contains the inverse image of the circle with centre (12, 5, 0) and radius 13.

7. Family of circles

Two circles are given in the Cartesian plane.

C_1: centre (3,4), radius 5, and

C_2: centre (2,1), radius 1.

7.1 Find the Hermitian matrices H_1 and H_2 that represent C_1 and C_2, respectively.

7.2 Find the cosine of the angle determined by H_1 and H_2.

7.3 Find an Hermitian matrix that represents the line in the pencil given by H_1 and H_2.

8. Orthogonal Circles

8.1 Let C_1 and C_2 be any 2 circles with centres O_1 and O_2, respectively. Let X be any point not on either circle, and not on either centre. Give a construction for a circle, K(X), passing through X that is orthogonal to both C_1 and C_2.

8.2 Suppose further that C_1 and C_2 are disjoint and that K(X) meets O_1O_2, the line of centres, in two points. Let Z be one of the intersection points. Let Σ be any circle whose centre is Z. Let C'_1 and C'_2 be the inverses with respect to Σ of C_1 and C_2, respectively. Prove that C'_1 and C'_2'are are concentric.