1. Two circles C_1 and C_2 are given by

$$C_1: \begin{bmatrix} 1 & 0 \\ 0 & 36 \end{bmatrix} \qquad C_2: \begin{bmatrix} 1 & -8+i \\ -8-i & 49 \end{bmatrix}$$

   (a) Find the matrix that represents every circle in the pencil determined by C_1 and C_2.

   (b) Find cosine of the angle determined by C_1 and C_2.

2. The sphere S in $\mathbb{R}^3$ is given by the equation $u^2 + v^2 + w^2 = 1$. Let S be the point with coordinates $(0,0,-1)$.

   (a) Find the equation of the stereographic projection from the plane $\{(x,y,0): x,y \in \mathbb{R}\}$ to S using the point S as the vertex of projection.

   (b) The circle $(x-8)^2 + (y-1)^2 = 16$ is mapped by the above projection to a circle on S. Find the equation of the plane containing that circle.

3. Let S be the sphere with equation $u^2 + v^2 + w^2 = 1$. Two planes
   $$\gamma_1: \quad 3u + v - w = 1$$
   $$\gamma_2: \quad u + 2v + 2w = 1$$

   (a) Find the poles P_1 and P_2 of the planes γ_1 and γ_2.

   (b) Find the line ℓ containing P_1 and P_2.

   (c) Show that the pole of any plane in the pencil determined by γ_1 and γ_2 lies on the line ℓ.

4.(a) Find the matrix of the collineation ϕ that maps the points with
       homogeneous coordinates (1,0,0), (0,1,0), (0,0,1), and (1,1,1) to
       (1,2,3), (1,1,0), (0,1,1), and (1,-1,4), respectively.

  (b) Let ℓ be the line with homogeneous coordinates [2,-1,1]. Find ℓ^{ϕ}.

5. Four collinear points in the plane are given with homogeneous coordinates
    A: (3,0,0), B: (1,2,-2), C: (8,1,-1), D: (0,-6,6).

    (a) Find the cross ratio (A,B;C,D).

    (b) Find the point X so that C and X are harmonic conjugates
         with respect to A and B.

6. (See next page.)

7. Find the equation of the non-degenerate conic that
        is tangent to the line [0,1,-1] at the point (1,1,1),
        is tangent to the line [1,0,0] at the point (0,0,1), and
        contains the point (2,-2,2).

6.(a) In the Desargues' Configuration below, find two triangles that

are perspective from the point A'. Find the line from which these

two trilaterals are perspective.
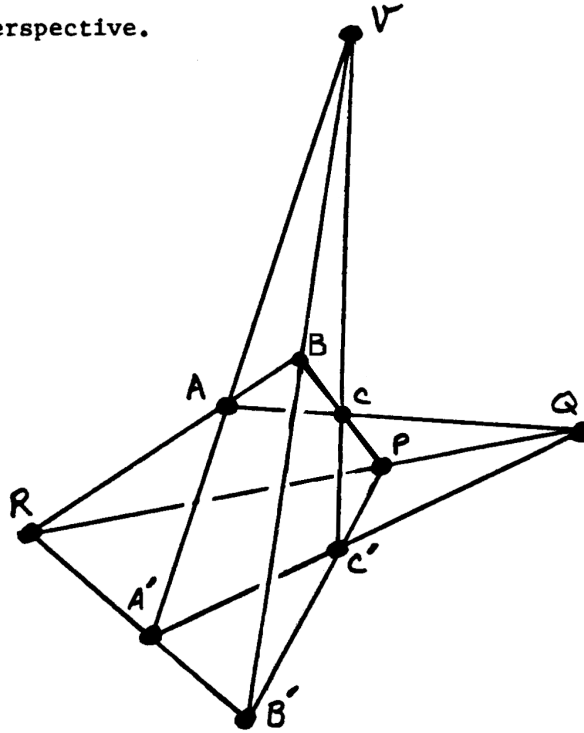
(b) The following homogeneous coordinates are given for points in the

figures above:

V: (1,1,1)   A: (1,0,0)   A': (3,1,1)

B: (0,1,0)   B': (2,1,2)

C: (0,0,1)   C': (4,4,5)

Find the coordinates of the line P,Q,R.