7 Constructing Inverses Part a

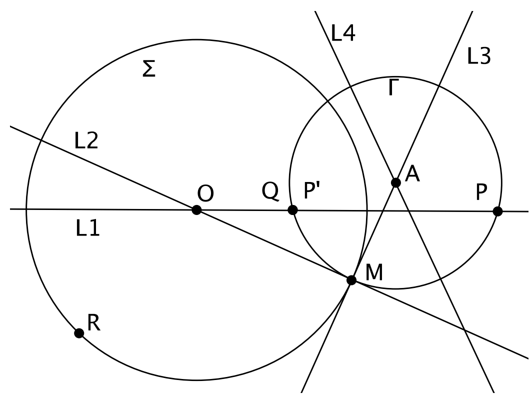
7.1 The inverse of a point

7.1.1 Construction using an orthogonal circle

Let O and R be distinct points and Σ be a circle with centre O and radius point R. Let P be any point distinct from O.

Consider this construction for the inverse of P with respect to Σ.

```
L1 = line ( O, P );
M  = point ( Σ\  L1 );
L2 = line ( O, M );
L3 = perp ( M, L2 );
L4 = pbis ( P, M );
A  = point ( L3, L4 );
Γ  = circle ( A, P );
Q  = point ( L1, Γ \ P ).
```



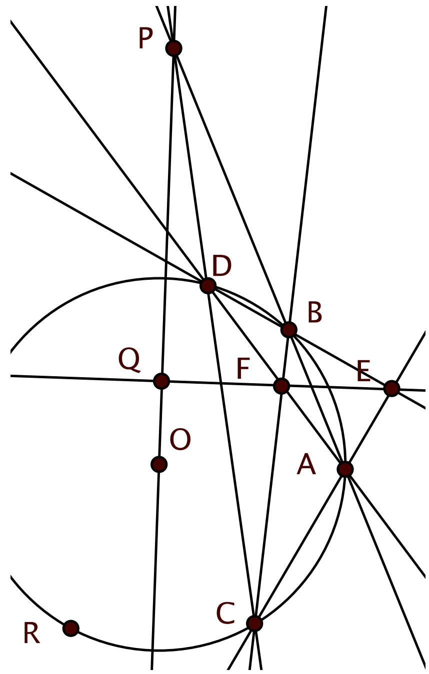
7.1.2 (*) Exploration

Confirm for yourself that when P is outside Σ, Q is inside, and vice versa. Confirm for yourself that Q seems not to move when you move M around Σ. Use the tool **Reflect Point about Circle** to find P', the inverse of P with respect to Σ. Use the **Move** tool to shift the *labels* of P' and Q slightly so that they do not overlap. Move the point P about and confirm that the *points* P' and Q continue to coincide. Find and use the tool **Relation between Two Objects** to compare P' and Q. When using this tool for two points that are close to each other, it is possible to select the individual points by clicking on their labels. (*) Submit your Figure and the Construction Protocol.

7.2 A straightedge only (no compass) construction

On a new figure, again let Σ be a circle with centre O, and let P be any point distinct from O. Consider this construction for Q:

```
L1 = line ( O, P );
A  = point ( Σ );
L2 = line ( P, A ) ;
B  = point ( Σ, L2 \A ) ;
C  = point ( Σ \L2 ) ;
L3 = line ( P, C ) ;
D  = point ( Σ, L3 \C ) ;
L4 = line ( A, C ) ;
L5 = line ( A, D ) ;
L6 = line ( B, C ) ;
L7 = line ( B, D ) ;
E  = point ( L4, L7 ) ;
F  = point ( L5, L6 ) ;
L8 = line ( E, F ) ;
Q  = point ( L1, L8 ).
```



7.2.1 (*) Exploration

Create your own copy of the figure, so that the line OP is more or less horizontal and P is to the right of the circle. Observe the positions of L_8 and Q, as you move A or C around Σ. (*) State your observations about the line L8 and Q as you move the points A and C.

7.2.2 (*) Simplification

(*) Create a new version of your figure for 7.2 by defining C so that it is either point of the intersection $\Sigma \cap L_1$ and D at the other. Again, by moving A about the circle Σ, observe the position of L_8 and Q. (*) State your observations about the points E and F, line L_8, and finally, the point Q. Submit your version the figure, its Construction Protocol and your observations.