SEMICROSSED PRODUCTS OF THE DISK ALGEBRA

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Abstract. If α is the endomorphism of the disk algebra, $\mathrm{A}(\mathbb{D})$, induced by composition with a finite Blaschke product b, then the semicrossed product $\mathrm{A}(\mathbb{D}) \times_\alpha \mathbb{Z}^+$ imbeds canonically, completely isometrically into $\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+$. Hence in the case of a non-constant Blaschke product b, the C*-envelope has the form $\mathrm{C}(\mathcal{S}_b) \times_s \mathbb{Z}$, where $(\mathcal{S}_b, s)$ is the solenoid system for $(\mathbb{T}, b)$. In the case where b is a constant, then the C*-envelope of $\mathrm{A}(\mathbb{D}) \times_\alpha \mathbb{Z}^+$ is strongly Morita equivalent to a crossed product of the form $\mathrm{C}_0(\mathcal{S}_e) \times_s \mathbb{Z}$, where $e \colon \mathbb{T} \times \mathbb{N} \longrightarrow \mathbb{T} \times \mathbb{N}$ is a suitable map and $(\mathcal{S}_e, s)$ is the solenoid system for $(\mathbb{T} \times \mathbb{N}, e)$.

1. Introduction

If $\mathcal{A}$ is a unital operator algebra and α is a completely contractive endomorphism, the semicrossed product is an operator algebra $\mathcal{A} \times_\alpha \mathbb{Z}_+$ which encodes the covariant representations of $(\mathcal{A}, \alpha)$: namely completely contractive unital representations $\rho : \mathcal{A} \to \mathcal{B}(\mathcal{H})$ and contractions T satisfying

$$\rho(a)T = T\rho(\alpha(a)) \quad \text{for all } a \in \mathcal{A}.$$

Such algebras were defined by Peters **[9]** when $\mathcal{A}$ is a C*-algebra.

One can readily extend Peter's definition **[9]** of the semicrossed product of a C*-algebra by a $*$-endomorphism to unital operator algebras and unital completely contractive endomorphisms. One forms the *polynomial algebra* $\mathcal{P}(\mathcal{A}, \mathfrak{t})$ of formal polynomials of the form $p = \sum_{i=0}^n \mathfrak{t}^i a_i$, where $a_i \in \mathcal{A}$, with multiplication determined by the covariance relation $a\mathfrak{t} = \mathfrak{t}\alpha(a)$ and the norm

$$\|p\| = \sup_{(\rho,T) \text{ covariant}} \Big\| \sum_{i=0}^n T^i \rho(a_i) \Big\|.$$

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This supremum is clearly dominated by $\sum_{i=0}^{n} \|a_i\|$; so this norm is well defined. The completion is the semicrossed product $\mathcal{A} \times_\alpha \mathbb{Z}_+$. Since this is the supremum of operator algebra norms, it is also an operator algebra norm. By construction, for each covariant representation (ρ, T), there is a unique completely contractive representation $\rho \times T$ of $\mathcal{A} \times_\alpha \mathbb{Z}_+$ into $\mathcal{B}(\mathcal{H})$ given by

$$\rho \times T(p) = \sum_{i=0}^{n} T^i \rho(a_i).$$

This is the defining property of the semicrossed product.

In this note, we examine semicrossed products of the disk algebra by an endomorphism which extends to a $*$-endomorphism of $\mathrm{C}(\mathbb{T})$. In the case where the endomorphism is injective, these have the form $\alpha(f) = f \circ b$ where b is a non-constant Blaschke product. We show that every covariant representation of $(\mathrm{A}(\mathbb{D}), \alpha)$ dilates to a covariant representation of $(\mathrm{C}(\mathbb{T}), \alpha)$. This is readily dilated to a covariant representation (σ, V), where σ is a $*$-representation of $\mathrm{C}(\mathbb{T})$ (so $\sigma(z)$ is unitary) and V is an isometry. To go further, we use the recent work of Kakariadis and Katsoulis [**6**] to show that $\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+$ imbeds completely isometrically into a C*-crossed product $\mathrm{C}(\mathcal{S}_b) \times_\alpha \mathbb{Z}$. In fact,

$$\mathrm{C}^*_{\mathrm{e}}(\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+) = \mathrm{C}(\mathcal{S}_b) \times_\alpha \mathbb{Z},$$

and as a consequence, we obtain that (ρ, T) dilates to a covariant representation (τ, W), where τ is a $*$-representation of $\mathrm{C}(\mathbb{T})$ (so $\sigma(z)$ is unitary) and W is a unitary.

In contrast, if α is induced by a constant Blashcke product, we can no longer identify $\mathrm{C}^*_{\mathrm{e}}(\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+)$ up to isomorphism. In that case, α is evaluation at a boundary point. Even though every covariant representation of $(\mathrm{A}(\mathbb{D}), \alpha)$ dilates to a covariant representation of $(\mathrm{C}(\mathbb{T}), \alpha)$, the theory of [**6**] is not directly applicable since α is not injective. Instead, we use the process of "adding tails to C*-correspondences" [**8**], as modified in [**3, 7**] and we identify $\mathrm{C}^*_{\mathrm{e}}(\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+)$ up to strong Morita equivalence as a crossed product. In Theorem 2.6 we show that $\mathrm{C}^*_{\mathrm{e}}(\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+)$ is strongly Morita equivalent to a C*-algebra of the form $\mathrm{C}_0(\mathcal{S}_e) \times_s \mathbb{Z}$, where

$$e \colon \mathbb{T} \times \mathbb{N} \longrightarrow \mathbb{T} \times \mathbb{N}$$

is a suitable map and $(\mathcal{S}_e, s)$ is the solenoid system for $(\mathbb{T} \times \mathbb{N},\, e)$.

Semi-crossed products of the the disc algebra were introduced and first studied by Buske and Peters in [**1**], following relevant work of Hoover, Peters and Wogen [**5**]. The algebras $\mathrm{A}(\mathbb{D}) \times_\alpha \mathbb{Z}^+$, where α is an arbitrary endomorphism, where classified up to algebraic endomorphism in [**2**]. Results associated with their C*-envelope can be found in [**1**, Proposition III.13] and [**10**, Theorem 2]. The results of the present paper subsume and extend these earlier results.

2. The Disk Algebra

The C*-envelope of the disk algebra $\mathrm{A}(\mathbb{D})$ is $\mathrm{C}(\mathbb{T})$, the space of continuous functions on the unit circle. Suppose that α is an endomorphism of $\mathrm{C}(\mathbb{T})$ which leaves $\mathrm{A}(\mathbb{D})$ invariant. We refer to the restriction of α to $\mathrm{A}(\mathbb{D})$ as α as well. Then $b = \alpha(z) \in \mathrm{A}(\mathbb{D})$; and has spectrum

$$\sigma_{\mathrm{A}(\mathbb{D})}(b) \subset \sigma_{\mathrm{A}(\mathbb{D})}(z) = \overline{\mathbb{D}}$$

and

$$\sigma_{\mathrm{C}(\mathbb{T})}(b) \subset \sigma_{\mathrm{C}(\mathbb{T})}(z) = \mathbb{T}.$$

Thus $\|b\| = 1$ and $b(\mathbb{T}) \subset \mathbb{T}$. It follows that b is a finite Blaschke product. Therefore $\alpha(f) = f \circ b$ for all $f \in \mathrm{C}(\mathbb{T})$. When b is not constant, α is completely isometric.

A (completely) contractive representation ρ of $\mathrm{A}(\mathbb{D})$ is determined by $\rho(z) = A$, which must be a contraction. The converse follows from the matrix von Neumann inequality; and shows that $\rho(f) = f(A)$ is a complete contraction. A covariant representation of $(\mathrm{A}(\mathbb{D}), \alpha)$ is thus determined by a pair of contractions (A, T) such that

$$AT = Tb(A).$$

The representation of $\mathrm{A}(\mathbb{D}) \times_\alpha \mathbb{Z}^+$ is given by

$$\rho \times T\Big(\sum_{i=0}^{n} \mathfrak{t}^i f_i\Big) = \sum_{i=0}^{n} T^i f_i(A),$$

which extends to a completely contractive representation of the semicrossed product by the universal property.

A contractive representation σ of $\mathrm{C}(\mathbb{T})$ is a $*$-representation, and is likewise determined by $U = \sigma(z)$, which must be unitary; and all unitary operators yield such a representation by the functional calculus. A covariant representation of $(\mathrm{C}(\mathbb{T}), \alpha)$ is given by a pair (U, T) where U is unitary and T is a contraction satisfying $UT = Tb(U)$. To see this, multiply on the left by U^* and on the right by $b(U)^*$ to obtain the identity

$$U^*T = Tb(U)^* = T\bar{b}(U) = T\alpha(\bar{z})(U).$$

The set of functions $\{f \in \mathrm{C}(\mathbb{T}) : f(U)T = T\alpha(f)(U)\}$ is easily seen to be a norm closed algebra. Since it contains z and $\bar{z}$, it is all of $\mathrm{C}(\mathbb{T})$. So the covariance relation holds.

Theorem 2.1. *Let b be a finite Blaschke product, and let $\alpha(f) = f \circ b$. Then $\mathrm{A}(\mathbb{D}) \times_\alpha \mathbb{Z}^+$ is (canonically completely isometrically isomorphic to) a subalgebra of $\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+$.*

Proof. To establish that $\mathrm{A}(\mathbb{D}) \times_\alpha \mathbb{Z}^+$ is completely isometric to a subalgebra of $\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+$, it suffices to show that each (A, T) with $AT = Tb(A)$ has a dilation to a pair (U, S) with U unitary and S a contraction such that

$$US = Sb(U) \quad \text{and} \quad P_{\mathcal{H}} S^n U^m|_{\mathcal{H}} = T^n A^m \text{ for all } m, n \geq 0.$$

This latter condition is equivalent to $\mathcal{H}$ being semi-invariant for the algebra generated by U and S.

The covariance relation can be restated as

$$\begin{bmatrix} A & 0 \\ 0 & b(A) \end{bmatrix} \begin{bmatrix} 0 & T \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & T \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A & 0 \\ 0 & b(A) \end{bmatrix}$$

Dilate A to a unitary U which leaves $\mathcal{H}$ semi-invariant. Then $\left[\begin{smallmatrix} A & 0 \\ 0 & b(A) \end{smallmatrix}\right]$ dilates to $\left[\begin{smallmatrix} U & 0 \\ 0 & b(U) \end{smallmatrix}\right]$. By the Sz.Nagy-Foiaş Commutant Lifting Theorem, we may dilate $\left[\begin{smallmatrix} 0 & T \\ 0 & 0 \end{smallmatrix}\right]$ to a contraction of the form $\left[\begin{smallmatrix} * & S \\ * & * \end{smallmatrix}\right]$ which commutes with $\left[\begin{smallmatrix} U & 0 \\ 0 & \alpha(U) \end{smallmatrix}\right]$ and has $\mathcal{H} \oplus \mathcal{H}$ as a common semi-invariant subspace. Clearly, we may take the $*$ entries to all equal 0 without changing things. So (U, S) satisfies the same covariance relations $US = Sb(U)$. Therefore we have obtained a dilation to the covariance relations for $(\mathrm{C}(\mathbb{T}), \alpha)$. ∎

Once we have a covariance relation for $(\mathrm{C}(\mathbb{T}), \alpha)$, we can try to dilate further. Extending S to an isometry V follows a well-known path. Observe that

$$b(U)S^*S = S^*US = S^*Sb(U).$$

Thus $D = (I - S^*S)^{1/2}$ commutes with $b(U)$. Write $b^{(n)}$ for the composition of b with itself n times, Hence we can now use the standard Schaeffer dilation of S to an isometry V and simultaneously dilate U to U_1 as follows:

$$V = \begin{bmatrix} S & 0 & 0 & 0 & \dots \\ D & 0 & 0 & 0 & \dots \\ 0 & I & 0 & 0 & \dots \\ 0 & 0 & I & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots & \ddots \end{bmatrix} \text{ and } U_1 = \begin{bmatrix} U & 0 & 0 & 0 & \dots \\ 0 & b(U_1) & 0 & 0 & \dots \\ 0 & 0 & b^{(2)}(U_1) & 0 & \dots \\ 0 & 0 & 0 & b^{(3)}(U_1) & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

A simple calculation shows that $U_1 V = V b(U_1)$. So as above, (U, V) satisfies the covariance relations for $(\mathrm{C}(\mathbb{T}), \alpha)$.

We would like to make V a unitary as well. This is possible in the case where b is non-constant, but the explicit construction is not obvious. Instead, we use the theory of C*-envelopes and maximal dilations. First we need the following.

Lemma 2.2. *Let b be a finite Blaschke product, and let $\alpha(f) = f \circ b$. Then*

$$\mathrm{C}^*_e(\mathrm{A}(\mathbb{D}) \times_\alpha \mathbb{Z}^+) \simeq \mathrm{C}^*_e(\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+).$$

Proof. The previous Theorem identifies $\mathrm{A}(\mathbb{D}) \times_\alpha \mathbb{Z}^+$ completely isometrically as a subalgebra of $\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+$. The C*-envelope $\mathcal{C}$ of $\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+$ is a Cuntz-Pimsner algebra containing a copy of $\mathrm{C}(\mathbb{T})$ which is invariant under gauge actions. Now $\mathcal{C}$ is a C*-cover of $\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+$, so it is easy to see that it is also a C*-cover of $\mathrm{A}(\mathbb{D}) \times_\alpha \mathbb{Z}^+$. Since $\mathrm{A}(\mathbb{D}) \times_\alpha \mathbb{Z}^+$ is invariant under the same gauge actions, its Shilov ideal $\mathcal{J} \subseteq \mathcal{C}$ will be invariant by these actions as well. If $\mathcal{J} \neq 0$ then by gauge invariance $\mathcal{J} \bigcap \mathrm{C}(\mathbb{T}) \neq 0$. Since the quotient map

$$\mathrm{A}(\mathbb{D}) \longrightarrow \mathrm{C}(\mathbb{T})/(\mathcal{J} \cap \mathrm{C}(\mathbb{T}))$$

is completely isometric, we obtain a contradiction. Hence $\mathcal{J} = 0$ and the conclusion follows. ∎

We now recall some of the theory of semicrossed products of C*-algebras. When $\mathfrak{A}$ is a C*-algebra, the completely isometric endomorphisms are the faithful $*$-endomorphisms. In this case, Peters shows [**9**, Prop.I.8] that there is a unique C*-algebra $\mathfrak{B}$, a $*$-automorphism β of $\mathfrak{B}$ and an injection j of $\mathfrak{A}$ into $\mathfrak{B}$ so that $\beta \circ j = j\alpha$ and $\mathfrak{B}$ is the closure of $\bigcup_{n \geq 0} \beta^{-n}(j(\mathfrak{A}))$. It follows [**9**, Prop.II.4] that $\mathfrak{A} \times_\alpha \mathbb{Z}_+$ is completely isometrically isomorphic to the subalgebra of the crossed product algebra $\mathfrak{B} \times_\beta \mathbb{Z}$ generated as a non-self-adjoint algebra by an isomorphic copy $j(\mathfrak{A})$ of $\mathfrak{A}$ and the unitary $\mathfrak{u}$ implementing β in the crossed product. Actually, Kakariadis and the second author [**6**, Thm.2.5] show that $\mathfrak{B} \times_\beta \mathbb{Z}$ is the C*-envelope of $\mathfrak{A} \times_\alpha \mathbb{Z}_+$. This last result is valid for semicrossed products of not necessarily unital C*-algebras by (non-degenerate) injective $*$-endomorphisms and we will use it below in that form.

In the case where $\mathfrak{A} = \mathrm{C}_0(X)$ is commutative and α is induced by an injective self-map of X, the pair $(\mathfrak{B}, \beta)$ has an alternative description.

Definition 2.3. Let X be a topological space and φ a surjective self-map of X. We define the *solenoid system of* (X, φ) to be the pair $(\mathcal{S}_\varphi, s)$, where

$$\mathcal{S}_\varphi = \{(x_n)_{n \geq 1} : x_n = \varphi(x_{n+1}), x_n \in X, n \geq 1\}$$

equipped with the relative topology inherited from the product topology on $\prod_{i=1}^{\infty} X_i$, $X_i = X$, $i = 1, 2, \dots$, and s is the backward shift on $\mathcal{S}_\varphi$.

It is easy to see that if $\mathfrak{A} = \mathrm{C}_0(X)$, with X a locally compact Hausdorff space, and α is induced by an surjective self-map φ of X, then the pair $(\mathfrak{B}, \beta)$ for $(\mathfrak{A}, \alpha)$ described above, is conjugate to the solenoid system $(\mathcal{S}_\varphi, s)$. Therefore, we obtain

Corollary 2.4. *Let b be a non-constant finite Blaschke product, and let $\alpha(f) = f \circ b$ on $\mathrm{C}(\mathbb{T})$. Then*

$$\mathrm{C}^*_e(\mathrm{A}(\mathbb{D}) \times_\alpha \mathbb{Z}^+) = \mathrm{C}(\mathcal{S}_b) \times_s \mathbb{Z}.$$

where $(\mathcal{S}_b, s)$ is the solenoid system of $(\mathbb{T}, b)$.

The above Theorem leads to a dilation result, for which a direct proof is far from obvious.

Corollary 2.5. *Let α be an endomorphism of $\mathrm{A}(\mathbb{D})$ induced by a non-constant finite Blaschke product and let $A, T \in \mathcal{B}(\mathcal{H})$ be contractions satisfying*

$$AT = T\alpha(A).$$

Then there exist unitary operators U and W on a Hilbert space $\mathcal{K} \supset \mathcal{H}$ which simultaneously dilate A and T, in the sense that

$$P_\mathcal{H} W^m U^n|_\mathcal{H} = T^m A^n \quad \textit{for all } m, n \geq 0,$$

so that

$$UW = W\alpha(U).$$

Proof. Every covariant representation (A, T) of $(\mathrm{A}(\mathbb{D}), \alpha)$ dilates to a covariant representation (U_1, V) of $(\mathrm{C}(\mathbb{T}), \alpha)$. This in turn dilates to a maximal dilation τ of $\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+$, in the sense of Dritschel and McCullough [**4**]. The maximal dilations extend to $*$-representations of the C*-envelope. Then A is dilated to $\tau(j(z)) = U$ is unitary and T dilates to the unitary W which implements the automorphism β on $\mathfrak{B}$, and restricts to the action of α on $\mathrm{C}(\mathbb{T})$. ∎

The situation changes when we move to *non-injective* endomorphisms α of $\mathrm{A}(\mathbb{D})$. Indeed, let $\lambda \in \mathbb{T}$ and consider the endomorphism α_λ of $\mathrm{A}(\mathbb{D})$ induced by evaluation on λ, i.e.,

$$\alpha_\lambda(f)(z) = f(\lambda) \quad \text{for all } z \in \mathbb{D}.$$

(Thus α_λ is the endomorphism of $\mathrm{A}(\mathbb{D})$ corresponding to a constant Blaschke product.) If two contractions A, T satisfy

$$AT = T\alpha_\lambda(A) = \lambda T,$$

then the existence of unitary operators U, W, dilating A and T respectively, implies that $A = \lambda I$. It is easy to construct a pair A, T satisfying $AT = \lambda T$ and yet $A \neq \lambda I$. This shows that the analogue Corollary 2.5 fails for $\alpha = \alpha_\lambda$ and therefore one does not expect $\mathrm{C}^*_{\mathrm{e}}(\mathrm{A}(\mathbb{D}) \times_{\alpha_\lambda} \mathbb{Z}^+)$ to be isomorphic to the crossed product of a commutative C*-algebra, at least under canonical identifications. However as we have seen, a weakening of Corollary 2.5 is valid for $\alpha = \alpha_\lambda$ if one allows W to be an isometry instead of a unitary operator. In addition, we can identify $\mathrm{C}^*_{\mathrm{e}}(\mathrm{A}(\mathbb{D}) \times_\alpha \mathbb{Z}^+)$ as being *strongly Morita equivalent* to a crossed product C*-algebra. Indeed, if

$$e \colon \mathbb{T} \times \mathbb{N} \longrightarrow \mathbb{T} \times \mathbb{N}$$

is defined as

$$e(z, n) = \begin{cases} (1, 1) & \text{if } n = 1 \\ (z, n-1) & \text{otherwise,} \end{cases}$$

then

Theorem 2.6. *Let $\alpha = \alpha_\lambda$ be an endomorphism of* $\mathrm{A}(\mathbb{D})$ *induced by evaluation at a point $\lambda \in \mathbb{T}$. Then $\mathrm{C}^*_e(\mathrm{A}(\mathbb{D}) \times_\alpha \mathbb{Z}^+)$ is strongly Morita equivalent to $\mathrm{C}_0(\mathcal{S}_e) \times_s \mathbb{Z}$, where $e \colon \mathbb{T} \times \mathbb{N} \longrightarrow \mathbb{T} \times \mathbb{N}$ is defined above and $(\mathcal{S}_e, s)$ is the solenoid system of* $(\mathbb{T} \times \mathbb{N}, e)$.

Proof. In light of Lemma 2.2, it suffices to identify the C*-envelope of $\mathrm{C}(\mathbb{T}) \times_\alpha \mathbb{Z}^+$. As α is no longer an injective endomorphism of $\mathrm{C}(\mathbb{T})$, we invoke the process of adding tails to C*-correspondences [**8**], as modified in [**3, 7**].

Indeed, [**7**, Example 4.3] implies that the C*-envelope of the tensor algebra associated with the dynamical system $(\mathrm{C}(\mathbb{T}), \alpha)$ is strongly Morita equivalent to the Cuntz-Pimsner algebra associated with the injective dynamical system $(\mathbb{T} \times \mathbb{N}, e)$ defined above. Therefore by invoking the solenoid system of $(\mathbb{T} \times \mathbb{N}, e)$, the conclusion follows from the discussion following Lemma 2.2. ∎

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