PM 950 **Assignment 2** **Due Tuesday October 21**

1. (a) If K is a p-set for A and $L \subset K$ is a p-set for $A|_K$, prove L is a p-set for A.

   (b) Show that every p-set with at least 2 points contains a proper subset which is a p-set. Hence show that every p-set contains a p-point.

2. Let A be a function algebra, and let Γ be its Shilov boundary.

   (a) Let $\varphi \in \mathcal{M}_A$. Prove that there is a minimal closed subset E of Γ supporting a representing measure for φ.

   (b) Suppose that $\varphi \in \mathcal{M}_A \setminus \Gamma$, and let E be a minimal closed support set found in (a). Prove that E has no isolated points.
   Hint: if $x \in E$ is isolated, pick $f \in A$ with $f(x) = 0$ and $\varphi(f) = 1$.

3. Let A_α be the algebra from Assignment 1. Suppose that B is a function algebra so that $A_\alpha \subset B \subset \mathrm{C}(\mathbb{T}^2)$. Show that either $B = A_\alpha$ or $B = \mathrm{C}(\mathbb{T}^2)$.
   Hint: Either z is invertible in B or there is a $\varphi \in \mathcal{M}_B$ so that $\varphi(z) = 0$. Look up a proof of Wermer's maximality theorem, and adapt the argument. See Gamelin, p.38, or Hoffman, *Banach spaces of analytic functions*, p.93.

4. (a) Let $f \in A$ and $\varphi \in \mathcal{M}_A$. Show that if $|\varphi(f)| = \|f\|$, then f is constant on the Gleason part $P(\varphi)$.

   (b) Let K be a compact subset of a Gleason part P. Prove that if $\psi \in \mathcal{M}_A$ satisfies $|\psi(f)| \le \|f\|_K$ for all $f \in A$, then $\psi \in P$.

5. Let $\varphi \in \mathcal{M}_A$. Suppose that there is a function $g \in \ker\varphi$ which has an nth root $g_n \in A$ for all $n \ge 1$. Prove that $\hat{g}$ vanishes on the Gleason part of φ.

6. Let $T : A \to A$ be a surjective isometry.

   (a) Show that there is a bijection σ of $\mathrm{Ch}(A)$ onto itself and a map $\lambda : \mathrm{Ch}(A) \to \mathbb{T}$ so that $T^*\delta_x = \lambda(x)\delta_{\sigma(x)}$ for all $x \in \mathrm{Ch}(A)$.

   (b) Show that $\lambda = T(1)$ restricted to $\mathrm{Ch}(A)$.

   (c) Show that $\lambda(x)T(fg)(x) = T(f)(x)\,T(g)(x)$ for $x \in \mathrm{Ch}(A)$. Deduce that λ is invertible in A. **Hint:** choose f, g to make the right side nice.

   (d) Show that $|\lambda(x)| = 1$ on all of $\mathcal{M}_A$, and that $T(f) = \lambda\theta(f)$, where θ is an isometric automorphism of A.