PM 810 Possible Talks

I plan to schedule the talks for the days of April 8–11. If you have an exam one of those days, let me know as soon as possible so that I can avoid conflicts. You are expected to attend all of the talks, and missed talks will count against you.

The purpose of this project is to read a short paper or sections from a book on a topic related to the course. It should not be something that you learned in another course. I want a short write-up of roughly 6 to 10 pages including references to all sources used. In addition, there will be a *25 minute* presentation. This is a short time, so you will have to summarize the topic. Concentrate on definitions, examples, statements of results, concepts and key ideas rather than attempting to give a complete proof. The paper should include these things plus the proofs *in your own words—do not just copy—fill in details to clarify the arguments.* There is no need to do everything in the paper or book section.

The list below are possible topics, but if you have other ideas for a project, all you have to do is clear the idea with me. They are not in any particular order except for the rough grouping. Once you have chosen a topic. Let me know because only one person can do a particular item—and it is first come, first served.

Banach Algebras

1. For a compact subset $X \subset \mathbb{C}^n$, there are algebras $P(X)$ and $R(X)$ generated by the polynomials and rational functions with no poles in X, respectively. See Kaniuth, *A course in commutative Banach algebras*, section 2.5.
2. The Shilov boundary of a commutative Banach algebra. See Kaniuth, *A course in commutative Banach algebras*, section 3.3.
3. Singer–Wermer Theorem on derivations of a commutative Banach algebra. See Bonsall and Duncan, *Complete normed algebras*, section II.18, Theorem 9.
4. Cohen's factorization theorem: if A has an approximate identity and X is a Banach A-module, then $X = AX$. Bonsall and Duncan, section 1.11.
5. Shilov's idempotent theorem: if X is a clopen subset of $\mathcal{M}(A)$ for a commutative Banach algebra, then A contains an idempotent e such that $\hat{e} = \chi_X$. See Kaniuth, *A course in commutative Banach algebras*, section 3.5. This requires a multivariable functional calculus.
6. Proper closed ideals of $C^n[0,1]$. See Kaniuth, *A course in commutative Banach algebras*, section 5.3.
7. The Arens product on the second dual of a Banach algebra and Arens regularity. Operator algebras are Arens regular. Blecher and LeMerdy, *Operator algebras and their modules*, section 2.5 and A.5.5-5.7.
8. The trace class operators are the dual of the compact operators and the predual of $\mathcal{B}(\mathcal{H})$. Possible sources include my *Nest Algebras* and J. Conway's *A course in operator theory.*

C*-Algebras

9. C*-algebras of compact operators. See *C*-algebras by Example*, section I.10.
10. UHF algebras. See *C*-algebras by Example*, section III.1–III.3. Work through these sections for the simplified case of UHF algebras leading to Theorem III.5.2.
11. Toeplitz operators. See *C*-algebras by Example*, section V.1.
12. The Weyl-von Neumann-Berg Theorem that every normal operator has a small compact perturbation which is diagonalizable. See *C*-algebras by Example*, section II.4.

13. The Cuntz algebra $\mathcal{O}_n$ is generated by n isometries with pairwise orthogonal range projections that sum to the identity. This uniquely determines $\mathcal{O}_n$ which is simple and purely infinite. See my *C*-algebras by Example*, section V.4-5.

14. The *irrational rotation algebra* is the C*-algebra generated by two unitaries U and V satisfying $VU = e^{2\pi i\theta}UV$ for irrational θ. It is simple and has a unique trace. See my *C*-algebras by Example*, section VI.1.

15. The C*-algebra of a group. See *C*-algebras by Example*, sections VII.1 and VII.4, say.

16. A $*$-homomorphism of one von Neumann algebra *onto* another is normal, and thus weak-$*$ continuous. See Dixmier, *Von Neumann algebras*, part 1, chapter 4, sections 2-3.

17. *C*-algebras of real rank zero*, L. Brown and G. Pedersen, J. Funct. Anal. **99** (1991), 131–149, Theorem 2.6.

18. Sakai's theorem: a C*-algebra is a von Neumann algebra if and only if it is a dual space. Pedersen, *C*-algebras and their automorphism groups*, section 3.9.

Nonself-adjoint Operator Algebras and Operator Theory

19. The radical of a nest algebra. J. Ringrose, *On some algebras of operators*, Proc. London Math. Soc. (3) **5** (1965), 61–83. See my book *Nest Algebras*, Thm.6.7.

20. W. Arveson, *Interpolation problems in nest algebras*, J. Func. Anal. **3** (1975), 208–233. The distance to nest algebras can be computed exactly. Also see my book *Nest Algebras*, Thm.9.5., for an easier proof.

21. W. Wogen, *Some counterexamples in nonselfadjoint algebras*, Ann. Math. **126** (1987), 415–427. A clever construction yields lots of operator algebras with unusual properties.

22. Lin's Theorem: two self-adjoint operators $A, B \in \mathcal{B}(H)$ such that $\|AB - BA\|$ is small are close to a commuting pair. See P. Friis and M. Rordam, *Almost commuting self-adjoint matrices—a short proof of Huaxin Lin's theorem*, J. Reine Angew. Math. **479** (1996), 121–131, section 2.

23. The universal operator algebra of a semigroup. D. Blecher and V. Paulsen, *Explicit construction of universal operator algebras and applications to polynomial factorization*, Proc. Amer. Math. Soc. **112** (1991), 839–850, section 2. This requires some knowledge from Vern's course in the fall.

24. J. Angelos, C. Cowen, and S. Narayan, *Triangular truncation and finding the norm of a Hadamard multiplier*, Lin. Alg. Appl. **170** (1992), 117–135. The operator sending a matrix to its upper triangular part has norm $\frac{\log n}{\pi}$ asymptotically.