Math 148 Review Guide

You should know all important defintions and the statements of all results that have names. In addition, you are responsible for knowing the proofs of the following results. I also give the corresponding result in Wade's book, although his statements and proofs are not always exactly the same as mine.

Riemann integration

(1) *Every continuous function on* $[a, b]$ *is Riemann integrable.* (Theorem 5.10)

(2) *The sum of two Riemann integrable functions is Riemann integrable.* (Theorem 5.19)

(3) *If* $f(x)$ *is continuous and* $G'(x) = f(x)$ *on* $[a, b]$*, then* $\int_a^b f(x)\,dx = G(b) - G(a)$.
I called this Corollary to FTC I. (It is easier than Theorem 5.28 (ii).)

Series

(4) The Comparison Test. (Theorem 6.14 is inadequate—he only deals with positive series.)
If $|b_n| \le a_n$ *for* $n \ge 1$*, then* (i) *if* $\sum_{n=1}^{\infty} a_n$ *converges, the* $\sum_{n=1}^{\infty} b_n$ *converges; and conversely,* (ii) *if* $\sum_{n=1}^{\infty} b_n$ *diverges, then* $\sum_{n=1}^{\infty} a_n$ *diverges.*

(5) Alternating Series Test. (Theorem 6.32—but his proof quoting Dirichlet's Theorem is not acceptable. You should use the direct (much easier) proof from class.)

(6) *If* $\sum_{n=1}^{\infty} a_n$ *converges absolutely, then every rearrangement converges to the same sum.* (Theorem 6.27)

Limits of functions

(7) *The uniform limit of a sequence of continuous functions on* $[a, b]$ *is continuous.* (Theorem 7.9)

(8) *The Integral Convergence Theorem.* (Theorem 7.10)

(9) The Weierstrass M-Test. (Theorem 7.15)

Techniques and results

Riemann integration: Riemann sums, upper and lower sums, Riemann's condition, FTC I, FTC II.
Integration: substitution, parts, recursion, partial fractions, other tricks. Improper integrals.
Applications: Volumes by disc method, shell method, by slices. Arc length, including parametric version. Area in polar coordinates.
Series tests: Comparison test, Integral test, Root test, Ratio test, Alternating series test, Dirichlet's test. Cauchy criterion. Absolute versus conditional convergence. Rearrangement theorem.
Series of functions: pointwise and uniform convergence, Cauchy criterion, sup norm, Integral Convergence Theorem, Weierstrass M-Test, completeness of $C[a, b]$.
Power series: Hadamard's Theorem, term-by-term differentiation and integration, Abel's theorem.
Taylor series: Taylor polynomials, Taylor's theorem on the error, big O, limits, approximations.
Differential equations: separation of variables, homogeneous of order zero, linear first order, linear constant coefficient, second order linear DEs and power series solutions.