steps of 0.1. This picture inspires one to look for a single equation satisfied by σ_1 and σ_2. In fact, it is not too hard to show that

$$(1-\sigma_1)\sigma_2 = \tfrac{1}{3}. \tag{11}$$

To see this, translate p so that $r_2 = 0$. Then $p'(x) = 3x^2 - 2(r_1 + r_3)x + r_1 r_3$. From this, we see that the product of the two roots of p' is $r_1 r_3/3$. However, the roots of p' are $(1-\sigma_1)r_1$ and $\sigma_2 r_3$.[1]

Figure 2 was produced in a similar manner but using quartic polynomials of the form $p(x) = x(x - r_2)(x - r_3)(x - r_4)$. The critical points were approximated by numerically solving the cubic equation $p'(x) = 0$ using Maple's **fsolve** procedure. This time Y_4 clearly appears to be a smooth surface in X_4.

REFERENCES

1. B. Anderson, Polynomial root dragging, *Amer. Math. Monthly* 100 (1993) 864–866.
2. R. Gelca, A short proof of a result on polynomials, *Amer. Math. Monthly* 100 (1993) 936–937.
3. L. F. Klosinski, G. L. Alexanderson and L. C. Larson, The fifty-second William Lowell Putnam mathematical competition, *Amer. Math. Monthly* 99 (1992), 715–724.
4. P. Walker, Separation of the zeros of polynomials, *Amer. Math. Monthly* 100 (1993) 272–273.

Department of Mathematics
Eastern Illinois University
Charleston, IL 61920
cfpga@eiu.edu

A Short Path to the Shortest Path

Peter D. Lax

This note contains a demonstration of the isoperimetric inequality. Our proof is somewhat simpler and more straightforward than the usual ones; it is eminently suitable for presentation in an honors calculus course.

1. *The Isoperimetric Inequality* says that a closed plane curve of length 2π encloses an area $\leq \pi$. Equality holds only for a circle.

Let $x(s)$, $y(s)$ be the parametric presentation of the curve, s arclength, $0 \leq s \leq 2\pi$. Suppose that we have so positioned the curve that the points $x(0)$, $y(0)$ and $x(\pi)$, $y(\pi)$ lie on the x-axis, i.e.

$$y(0) = 0 = y(\pi). \tag{1}$$

The area enclosed by the curve is given by the formula

$$A = \int_0^{2\pi} y\dot{x}\, ds, \tag{2}$$

where the dot denotes differentiation with respect to s. We write this integral as the sum $A_1 + A_2$ of an integral from 0 to π and from π to 2π, and show that each is $\leq \frac{\pi}{2}$.

[1]The author thanks the referee for this particularly nice derivation of (11).

According to a basic inequality,

$$ab \le \frac{a^2 + b^2}{2};$$

equality holds only when $a = b$. Applying this to $y = a$, $\dot{x} = b$, we get

$$A_1 = \int_0^{\pi} y\dot{x}\, ds \le \frac{1}{2}\int_0^{\pi} \left(y^2 + \dot{x}^2\right) ds. \tag{3}$$

Since s is arclength, $\dot{x}^2 + \dot{y}^2 = 1$; so we can rewrite (3) as

$$A_1 \le \frac{1}{2}\int_0^{\pi} \left(y^2 + 1 - \dot{y}^2\right) ds. \tag{3'}$$

Since $y = 0$ at $s = 0$ and π, we can factor y as

$$y(s) = u(s)\sin s, \tag{4}$$

u bounded and differentiable. Differentiate (4):

$$\dot{y} = \dot{u}\sin s + u\cos s.$$

Setting this into (3′) gives

$$A_1 \le \frac{1}{2}\int_0^{\pi} \left[u^2(\sin^2 s - \cos^2 s) - 2u\dot{u}\sin s\cos s - \dot{u}^2\sin^2 s + 1\right] ds. \tag{5}$$

The product $2u\dot{u}$ is the derivative of u^2; integrating by parts changes (5) into

$$A_1 \le \frac{1}{2}\int_0^{\pi} (1 - \dot{u}^2\sin^2 s)\, ds,$$

clearly $\le \pi/2$. Equality holds only if $\dot{u} \equiv 0$, which makes $y(s) \equiv$ constant $\sin s$. Since equality in (3) holds only if $y = \dot{x} = \sqrt{1 - \dot{y}^2}$, $y(s) \equiv \pm\sin s$, $x(s) \equiv \mp\cos s$ + constant. This is a semicircle. Q.e.d.

Courant Institute of Mathematical Sciences
New York University
251 Mercer Street
New York, NY 10012

A Note on Entire Solutions of the Eiconal Equation

Dmitry Khavinson

The eiconal equation $\sum_{i=1}^{n}(\partial u/\partial x_i)^2 = 1$, u: $\mathbf{R}^n \to \mathbf{R}$ is one of the main equations of geometrical optics. Its characteristics represent the light rays, while the level surfaces of solution u can be thought of as wave fronts (cf., e.g., [**3**]). Here,