Math 148 Assignment 2

Due 2:00 p.m. Friday, January 30 in the Math 148 dropbox.

1. Compute the following integrals:

   (a) $\displaystyle\int \frac{\sin^3 x}{\sqrt{\cos x}}\,dx$ (b) $\displaystyle\int x^2 \sin^{-1}(x^3)\,dx$ (c) $\displaystyle\int_0^{63} \frac{dt}{\sqrt{1+t}+\sqrt[3]{1+t}}$

2. Compute the following integrals:

   (a) $\displaystyle\int_1^2 \big(\log x\big)^2\,dx$ (b) $\displaystyle\int e^{2x}\cos(3x)\,dx.$ (c) $\displaystyle\int_{-1}^{1} x^3 e^{x^4}\cos 2x\,dx$

3. Compute the following integrals:

   (a) $\displaystyle\int \frac{5x^2-13x+9}{x^3-3x^2+4}\,dx$ (b) $\displaystyle\int_{-3}^{-2} \frac{x^2+8x+10}{(x^2+6x+10)^2}\,dx$ (c) $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{1}{5+\sin x+7\cos x}\,dx.$

4. (a) Compute a recursion formula for $I_m = \displaystyle\int x^a(\log x)^m\,dx$, $m \geq 0$ and $a \neq -1$.

   Hence obtain an explicit formula for I_3.

   (b) $\displaystyle\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$ **Hint:** Substitute $u = \pi - x$ and combine the two integrals.

5. Suppose that $f(x)$ is a C^2 function on $\mathbb{R}$ such that $|f(x)| \leq A$ and $|f''(x)| \leq C$ for $x \in \mathbb{R}$. Prove that $|f'(x)| \leq \sqrt{2AC}$.

   Hint: fix x_0 with $f'(x_0) = b \geq 0$. Get a lower bound for $f'(x_0 \pm h)$.
   Use this to estimate $\displaystyle\int_{x_0-H}^{x_0+H} f'(x)\,dx$ for a good choice of H.

6. Suppose that $f(0) = 0$ and $0 < f'(x) \leq 1$ for all $x \geq 0$. Show that

$$\int_0^x f(t)^3\,dt \leq \left(\int_0^x f(t)\,dt\right)^2 \qquad \text{for all } x > 0.$$

   When does equality hold?
   Hint: differentiate, factor and differentiate again.