Math 148 Assignment 2
Due 2:00 p.m. Friday, January 30 in the Math 148 dropbox.

1. Compute the following integrals:
   
   (a) \( \int \frac{\sin^3 x}{\sqrt{\cos x}} \, dx \)  
   (b) \( \int x^2 \sin^{-1}(x^3) \, dx \)  
   (c) \( \int_0^{63} \frac{dt}{\sqrt{1 + t + \sqrt{1 + t}}} \)

2. Compute the following integrals:
   
   (a) \( \int_1^2 (\log x)^2 \, dx \)  
   (b) \( \int e^{2x} \cos(3x) \, dx \)  
   (c) \( \int_{-1}^{1} x^3 e^x \cos 2x \, dx \)

3. Compute the following integrals:
   
   (a) \( \int \frac{5x^2 - 13x + 9}{x^3 - 3x^2 + 4} \, dx \)  
   (b) \( \int_{-3}^{-2} \frac{x^2 + 8x + 10}{(x^2 + 6x + 10)^2} \, dx \)  
   (c) \( \int_{-\pi/2}^{\pi/2} \frac{1}{5 + \sin x + 7 \cos x} \, dx \)

4. (a) Compute a recursion formula for \( I_m = \int x^a (\log x)^m \, dx \), \( m \geq 0 \) and \( a \neq -1 \).

   Hence obtain an explicit formula for \( I_3 \).

   (b) \( \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} \, dx \)  
       \textbf{Hint:} Substitute \( u = \pi - x \) and combine the two integrals.

5. Suppose that \( f(x) \) is a \( C^2 \) function on \( \mathbb{R} \) such that \( |f(x)| \leq A \) and \( |f''(x)| \leq C \) for \( x \in \mathbb{R} \).

   Prove that \( |f'(x)| \leq \sqrt{2AC} \).

   \textbf{Hint:} fix \( x_0 \) with \( f'(x_0) = b \geq 0 \). Get a lower bound for \( f'(x_0 \pm h) \).

   Use this to estimate \( \int_{x_0-H}^{x_0+H} f'(x) \, dx \) for a good choice of \( H \).

6. Suppose that \( f(0) = 0 \) and \( 0 < f'(x) \leq 1 \) for all \( x \geq 0 \). Show that

   \[ \int_0^x f(t)^3 \, dt \leq \left( \int_0^x f(t) \, dt \right)^2 \]  \text{ for all } x > 0.

   When does equality hold?

   \textbf{Hint:} differentiate, factor and differentiate again.