Preface

This book began with material for a two-semester course in functional analysis for graduate students. This material has been expanded in various ways during the writing of the book. This is followed by additional material on operator theory and operator algebras. A course in analysis on metric spaces and some knowledge of Lebesgue measure is assumed. More general measure theory is an asset, and certainly will be needed later in the book.

Part I is based on a one-semester course in functional analysis that I taught many times. Chapter 1 covers some necessary background in point-set topology. This is not a comprehensive treatment, but we will need to use nets and weak topologies. Normally I do not cover everything in this chapter until it is needed. Chapter 2 treats Banach spaces and establishes basic results such as the Banach–Steinhaus theorem, the Open Mapping theorem and the Hahn–Banach theorem. In the Chap. 3, topological vector spaces are introduced. Geometric versions of the Hahn–Banach theorem are established as well as the Krein–Milman and Krein–Smulian theorems. I do not usually cover all of the section on fixed point theorems nor weak compactness. They are included as a resource for some later material. The next two chapters deal with bounded linear operators between Banach spaces. In particular, the structure of compact operators and basic Fredholm theory is covered. Specializing to Hilbert spaces, the spectral theorem for compact normal operators is established.

Part II is based on our second-semester course, an introduction to Banach and C*-algebras. Chapter 6 covers the basic material on spectrum and parallels the treatment already given for operators. It then develops the Riesz–Dunford functional calculus, which is needed for operators as well. Chapter 7 deals with Gelfand theory for commutative Banach algebras, with extensive analysis of the examples of $L^1(G)$ and commutative C*-algebras. Chapter 8 deals with the representation theory of general Banach algebras. I usually don't have time for the Cohen factorization theorem. Chapter 9 is an introduction to the basics of C*-algebras, culminating in the Gelfand–Naimark theorem that every abstract C*-algebra has a faithful representation on Hilbert space. It is followed by a number of additional topics including the spectral theorem and group C*-algebras. I usually finish my course with the spectral theorem. Chapter 10 covers additional material on von Neumann algebras. This is an important subject, but there is no time for this in our syllabus.

Part III of the notes covers some specialized topics that I have covered in further courses. This is a range of topics in operator theory, dilation theory, nonself-adjoint operator algebras and noncommutative convexity. The treatment of Hardy spaces is chosen to provide tools needed to do operator theory on H^2, and to elucidate some of the nice Banach algebra properties of the disc algebra and H^∞. The chapter on Toeplitz operators was in large part inspired by Douglas's book [98].

The chapter on nest algebras takes some material from my out-of-print text [69], and contains a shorter, more direct proof of the Similarity theorem as well as Orr's interpolation theorem. Dilation theory is an important tool in operator theory and operator algebras. The subject of completely positive maps and its role in operator algebras is central to the modern theory. Our treatment was strongly influenced by Paulsen's book [226] and the treatise of Sz.-Nagy–Foiaş [297]. It has many applications. We provide a chapter with introductions to semicrossed products and multivariable operator theory, both commutative and non-commutative. The final chapter is an introduction to noncommutative convexity and non-commutative Choquet theory.

The bibliography goes back about a century. We have not referenced original papers earlier than that. This affects the Part I on functional analysis, as the later material begins with around 1929 with the spectral theorem for normal operators due independently to Stone and von Neumann. The notes and references in Dunford and Schwarz [103] provide additional references going back further.

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