SIAM J. APPL. MATH.
Vol. 0, No. 0, pp. 000–000

PASSIVITY OF MAGNETOSTRICTIVE MATERIALS*

SINA VALADKHAN†, KIRSTEN MORRIS‡, AND AMIR KHAJEPOUR†

Abstract. Magnetostrictive materials display large force and displacement in response to an applied field, as well as short response time. However, their nonlinear and hysteretic behavior has hindered their use. We prove, using the physics of the material, that these materials are passive. The corresponding energy storage function is shown to be the Helmholtz energy. This result is independent of the model used. The effect of varying load is included. Passivity is important because it can be used to obtain control systems that maintain stability despite uncertainties and disturbances. The minima of the storage function are also obtained. The storage function is written explicitly in the case of a common model for these materials, the Preisach model.

Key words. passivity, smart materials, Preisach model, magnetostrictive materials, stability

AMS subject classifications. 93D09, 93D25, 93D15, 93A30, 82D40

DOI. 10.1137/060651264

1. Introduction.

There has been a growing demand by industry in recent years for micropositioning devices. Micropositioning actuators are now frequently seen in scanning microscopes, chip manufacturing machines, biological cell micromanipulation and optical fiber alignment devices. Currently, many of these micropositioning tasks are done with piezoceramic actuators. Piezoceramic actuators exhibit almost linear behavior and have a reasonably fast response time.

Still, there is a demand for actuators with a larger stroke and faster response time. For this reason, the possibility of using other active materials for actuation is being examined. Terfenol-D, an alloy of iron, terbium, and dysprosium, has many advantages. Terfenol-D is a magnetostrictive material. Compared to other active materials, it has very large force and displacement with a short response time that makes it an attractive choice for actuation.

The use of magnetostrictive materials has been hindered by the fact that their response is highly nonlinear and hysteretic. Because of this nonlinearity, Terfenol-D actuators are difficult to control. In many micropositioning tasks, submicron accuracy is required. To achieve the required performance, actuators need to be used in a closed-loop feedback system. The controller in the feedback system must be able to stabilize the system under all conditions.

Dependence of the hysteresis on many physical conditions together with the nonlinear nature of the system make it difficult to establish stability for the closed loop. External physical conditions such as mechanical loading and temperature affect the behavior of magnetostrictive materials. Stability and performance of the control system must be maintained despite these system uncertainties and also despite disturbances. One of the most useful methods for showing stability of nonlinear systems is *passivity*. There are many passive physical systems [1]. Passive systems are important because the stability of closed-loop passive systems can be easily established. For

*Received by the editors January 30, 2006; accepted for publication (in revised form) October 20, 2006; published electronically DATE.
http://www.siam.org/journals/siap/x-x/65126.html
†Department of Mechanical Engineering, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada (svaladkh@uwaterloo.ca, akhajepour@uwaterloo.ca).
‡Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada (kmorris@uwaterloo.ca).

many nonlinear systems, this approach is the only way to show stability. Passivity has been used to obtain closed-loop stability for nonlinear systems in over 300 papers published in the last 10 years.

The Preisach model [2] is among the oldest models for magnetic materials. This model has been successfully applied to many hysteretic systems [3, 4]. In [5], the Preisach model is used with a set of ordinary differential equations to develop a rate-dependent hysteresis model. Open-loop stability and other properties of the model are discussed. These results are used to develop a model inverse-based controller for a magnetostrictive actuator [6].

In [7], an energy-based version of the Preisach model is introduced. Unlike the classical Preisach model, this model is based on a physical model for the material. In [3], it is shown that the Preisach operator is passive if the system output is the time-derivative of the output. The associated storage function is also computed. The result is applied to the control of a shape memory alloy actuator. In [8], this approach is extended to position control. The passivity results [3] are used in [9] to establish asymptotic stability of closed-loop systems containing hysteresis.

In the next section we give a brief review of standard material on passivity. It is subsequently shown, using physics, that magnetostrictive materials are passive. The storage function is identified to be the Helmholtz energy. No assumption on the model is used. The effects of varying load are included. The Preisach model is then introduced and the energy storage function is written explicitly using this model. The system equilibrium points are identified and discussed.

2. Passivity. In this section, passivity is defined in a dynamical systems framework. This framework will be used later for magnetostrictive materials. Consider a system with input $u \in U$, output $y \in U$, and state $x \in X$. The following is a standard definition for dynamical systems [1].

DEFINITION 1. *A dynamical system is defined through input, output and state spaces U and X, a readout operator r, and a state transition operator ϕ. The readout operator is a map from $U \times X$ to U. The state transition operator is a map from $\mathbb{R}^2 \times X \times U$ to X. The state transition operator must have the following properties for all $x_0 \in X$, t_0, t_1, $t_2 \in \mathbb{R}$, u, u_1, $u_2 \in U$:*

Consistency: $\phi(t_0, t_0, x_0, u) = x_0$.

Determinism: $\phi(t_1, t_0, x_0, u_1) = \phi(t_1, t_0, x_0, u_2)$ *for all* $t_1 \geq t_0$ *when* $u_1(t) = u_2(t)$ *for all* $t_0 \leq t \leq t_1$.

Semigroup: $\phi(t_2, t_0, x_0, u) = \phi(t_2, t_1, \phi(t_1, t_0, x_0, u), u)$ *when* $t_0 \leq t_1 \leq t_2$.

Stationarity: $\phi(t_1 + T, t_0 + T, x_0, u_T) = \phi(t_1, t_0, x_0, u)$ *for all* $t_1 \geq t_0$, $T \in \mathbb{R}$ *when* $u_T(t) = u(t + T)$ *for all* $t \in \mathbb{R}$.

DEFINITION 2 (see [1]). *Consider a dynamical system with state variables x, an input u, and output y. If there is a real-valued function $S(x)$ satisfying the following relation for any $t_i \leq t_f$ and if $S(x)$ is bounded from below, the dynamical system is called passive:*

$$S(x(t_i)) + \int_{t_i}^{t_f} \langle u, y \rangle \, dt \geq S(x(t_f)). \tag{1}$$

In this definition, $\langle ., . \rangle$ is the inner product on U. The variables u and y are vectors of the same dimension, so that $\langle u, y \rangle$ is defined. The scalar function $S(x)$ is called the storage function. Passive systems are frequently seen in engineering. The storage function is often the energy.

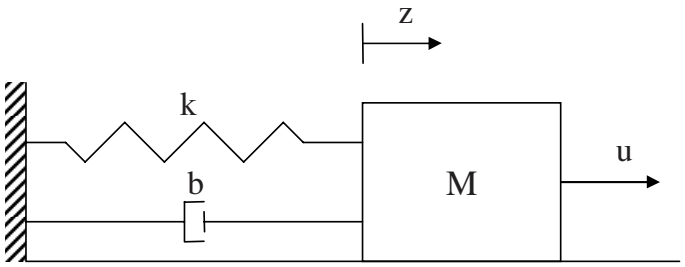

FIG. 1. *A spring-mass-dashpot system.*

Example. Consider a spring-mass-dashpot system (Figure 1). The following equation describes this system:

$$M\frac{d^2z}{dt^2} + b\frac{dz}{dt} + kz = u, \tag{2}$$

where u is the external force applied. The velocity of the mass $\dot{z}$ is considered to be the system output: $y = \dot{z}$. The state variables are z and $\dot{z}$. If both sides of (2) are multiplied by $\dot{z}$ and integrated from t_i to t_f, it becomes

$$\frac{M}{2}\left(\dot{z}^2(t_f) - \dot{z}^2(t_i)\right) + \int_{t_i}^{t_f} b\dot{z}^2 dt + \frac{k}{2}\left(z^2(t_f) - z^2(t_i)\right) = \int_{t_i}^{t_f} \langle u, y\rangle \, dt. \tag{3}$$

In this example, total energy is

$$E(z, \dot{z}) = \frac{1}{2}kz^2 + \frac{1}{2}M\dot{z}^2. \tag{4}$$

Using this definition, (3) can be rewritten as

$$E(z(t_i), \dot{z}(t_i)) + \int_{t_i}^{t_f} \langle u, y\rangle \, dt \geq E(z(t_f), \dot{z}(t_f)). \tag{5}$$

The storage function $E(z, \dot{z})$ is always nonnegative and, hence, bounded from below. As a result, this system is passive. When $u = 0$, the system goes to a state which minimizes E. The energy E is minimized when $z = 0$, $\dot{z} = 0$. This is the global system equilibrium point.

When the force applied to the system includes a constant force, such as gravity, its effect can be included in the system storage function. If the force applied to the mass is $F_{const} + u$, the following storage function is minimized at the equilibrium point:

$$\bar{E} = E - F_{const}z. \tag{6}$$

In this case, the equilibrium point is $z = \frac{F_{const}}{k}$, $\dot{z} = 0$.

Define the operator $\|.\|$ to be the Euclidean norm; that is, for any vector v, $\|v\|^2 = \langle v, v\rangle$. The following definitions are used to establish stability for the system [10, 11].

DEFINITION 3. *The set L_2 is the set of functions $x : \mathbb{R} \to \mathbb{R}^n$ for which the following expression is bounded:*

$$\int_0^\infty \|x(t)\|^2 \, dt < \infty. \tag{7}$$

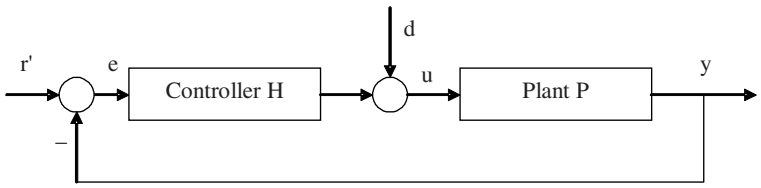

FIG. 2. *The standard feedback configuration.*

DEFINITION 4. *The set L_{2e} is the set of functions $x : \mathbb{R} \to \mathbb{R}^n$ for which the following expression is bounded for all $T \in \mathbb{R}$:*

$$\int_0^T \|x(t)\|^2 \, dt < \infty. \tag{8}$$

DEFINITION 5. *A mapping $R : L_{2e} \to L_{2e}$ is said to be L_2-stable if $x \in L_2$ implies that $Rx \in L_2$.*

Suppose that a given system P is passive. Consider the general feedback control configuration shown in Figure 2. If the controller H satisfies certain conditions, the following result can be used to show the stability of the controlled system.

THEOREM 6 (see [11, Theorem 10, p. 182]). *Consider the feedback system shown in Figure 2, where H and P map U to U. The set U is a subset of L_{2e}. Assume that for any r' and d in L_2 there are solutions e and u in L_{2e} and there are constants α_1, α_2, α_3, β_1, β_2, and β_3 such that for every real T and $x \in L_{2e}$ the following conditions hold:*

$$\begin{aligned} &\text{I} \quad \sqrt{\int_0^T \|Hx\|^2 \, dt} \le \alpha_1 \sqrt{\int_0^T \|x\|^2 \, dt} + \beta_1, \\ &\text{II} \quad \int_0^T \langle x, Hx \rangle \, dt \ge \alpha_2 \int_0^T \|x\|^2 \, dt + \beta_2, \\ &\text{III} \quad \int_0^T \langle Px, x \rangle \, dt \ge \alpha_3 \int_0^T \|Px\|^2 \, dt + \beta_3. \end{aligned} \tag{9}$$

If $\alpha_2 + \alpha_3 > 0$, then r, $d \in L_2$ implies that e, u, He, $y \in L_2$.

A passive system satisfies the third condition with $\alpha_3 = 0$ and $\beta_3 = \inf S(x) - S(x(0))$. The second and third conditions are similar to requiring that plant and controller be passive, but slightly stronger since $\alpha_2 + \alpha_3$ has to be strictly positive. The last line of the theorem states that the closed loop is L_2-stable.

This theorem can be used to establish stability for a large class of nonlinear systems. For many systems this theorem is the only way to establish stability. The passivity results which will be shown later can be used with this theorem to show stability for the magnetostrictive system.

3. Passivity for magnetostrictive materials. Since magnetostrictive materials dissipate energy, we expect them to be passive with some energy function as the storage function. In this section, the physical parameters of magnetostrictive materials are introduced. Three different energy functions for magnetostrictive materials and their suitability as a storage function are discussed. Finally, a proof of passivity is given.

Magnetostrictive materials react to a magnetic field. Suppose that a magnetostrictive sample is excited in a magnetic field produced by a coil. If there is an electrical

current in the coil, a nonzero magnetic field H is seen around the coil. Magnetic field H is a vector field, and it depends on the electrical current and the geometry. Magnetic field H acts on the magnetostrictive sample, and it is usually considered to be the input for the hysteretic system. As a result of this magnetic field, a magnetization M is seen in the material. The magnetization M is also a vector field, and it is considered to be the response or output of the hysteretic system. The relation between H and M depends on the material.

The magnetization M is not the only parameter affected by an external magnetic field H. The mechanical variables are also affected. For a material where the magnetic and mechanical responses are decoupled, the stress σ is usually considered to be the input for the mechanical part, and the strain ε, the response. For magnetostrictive materials, a magnetic field affects both magnetization and strain, and similarly for the stress. For magnetostrictive materials, generalized force and displacement are defined as follows:

$$F = \begin{pmatrix} \mu_0 H \\ \sigma \end{pmatrix}, \tag{10}$$

$$X = \begin{pmatrix} M \\ \varepsilon \end{pmatrix}. \tag{11}$$

Generalized force F is the system input and time-derivative of generalized displacement $\dot{X}$, the output. The constant μ_0 is a physical constant to ensure that $\mu_0 \langle H, M \rangle$ has the unit of energy per unit volume.

Various energy functions can be associated with magnetostrictive materials. Here these energy functions are introduced and their suitability as a storage function is discussed.

3.1. The internal energy. The internal energy U is the total potential energy stored in the material. The first law of thermodynamics holds for this energy function:

$$\frac{dU}{dt} = \frac{dQ}{dt} + \frac{dW}{dt}, \tag{12}$$

where $\frac{dQ}{dt}$ is the rate of thermal energy supplied to the material and $\frac{dW}{dt}$ is the rate of magnetic/mechanical work done on the system. The inequality of Clausius [12, p. 205] states that for any process $\frac{dS}{dt} \geq \frac{1}{T}\frac{dQ}{dt}$, where T is the temperature and S is the entropy. Using this inequality, the first law can be written as

$$\frac{dU}{dt} \leq T\frac{dS}{dt} + \frac{dW}{dt}. \tag{13}$$

A relation similar to the passivity inequality can be obtained by integrating both sides of (13) from t_i to t_f:

$$U_i + \int_{t_i}^{t_f} \left(T\frac{dS}{dt} + \frac{dW}{dt} \right) dt \geq U_f. \tag{14}$$

It is seen that thermal terms should appear in the system input/output; i.e., u should be $\binom{\mu_0 H}{\sigma}{T}$ and y should be $\binom{\dot{M}}{\dot{\varepsilon}}{\dot{S}}$. Since the energy stored in the material is limited, the amount of energy which can be pulled out of the material is also limited. This means that the energy function U has a lower bound. As a result, the internal energy U can be used as a storage function.

Thermal variables are usually difficult to work with, and for magnetostrictive materials they are difficult to measure. Extra thermal input and output are disadvantages to using internal energy as a storage function. For this reason, the internal energy is not chosen as the storage function.

3.2. The Gibbs energy. The following relation defines the Gibbs energy:

$$G = U - TS - \langle F, X \rangle . \tag{15}$$

Using the relation $\frac{dW}{dt} = \left\langle F, \frac{dX}{dt} \right\rangle$ and (10), (11) and (13), we obtain

$$\frac{dG}{dt} \leq -S\frac{dT}{dt} - \mu_0 \left\langle M, \frac{dH}{dt} \right\rangle - \varepsilon \frac{d\sigma}{dt}. \tag{16}$$

The Gibbs energy is a function of H. This means that H has to be included in the system states. This is awkward for several reasons. First, in this application H is an input. Second, consider a situation in which $\varepsilon = 0$ and H has a large value. The Gibbs energy can be made arbitrarily small by increasing H. This means that the Gibbs energy does not have a lower bound, and hence it is not a suitable storage function.

3.3. The Helmholtz energy. The Helmholtz free energy ψ is defined as

$$\psi = U - TS, \tag{17}$$

where T and S are the temperature and total entropy, respectively, of the system. Using the inequality of Clausius, the first law of thermodynamics can be written as

$$\frac{d\psi}{dt} \leq -S\frac{dT}{dt} + \frac{dW}{dt}. \tag{18}$$

Under constant temperature, this equation simplifies to

$$\frac{d\psi}{dt} \leq \frac{dW}{dt}. \tag{19}$$

This relation states that the work provided is more than the rate at which Helmholtz free energy is increased. It can be said that part of the work energy provided is absorbed by the system and added to the stored energy, while the rest is wasted in energy dissipation. It seems that the Helmholtz free energy is the energy actually stored in the system. In this respect, the Helmholtz energy is comparable to the energy storage function E in the mechanical example. Since the energy E is the storage function for the mechanical example, this comparison suggests the Helmholtz energy as the storage function. In the next subsection, a detailed proof of passivity, with the Helmholtz free energy as the storage function, is given.

3.4. Proof of passivity. It is assumed that, during any process discussed here, no phase transition occurs; for example, the material is not melting. This guarantees the existence of partial derivatives. All of the processes are under constant air pressure. Work done by the air pressure is neglected. For simplicity, from now on, it is also assumed that the thermal connection between the material and the surrounding environment is so good that the temperature of the material is always close to the room temperature T_0 and constant.

In a magnetic material, the ratio between the dipole magnetic energy and the energy of thermal fluctuations plays an important role. If the dipole magnetic energy

is small compared to thermal fluctuations, the material is called *paramagnetic*. In this case, the dipoles are mostly affected by thermal fluctuations and the external magnetic field H. Dipole-dipole interaction is weak. Because of thermal fluctuations, paramagnetic materials are memoryless and have no hysteresis. On the other hand, if the dipole magnetic energy is large compared to thermal fluctuations, the material is called *ferromagnetic*. Dipoles in a ferromagnetic sample retain their state, and the material has memory. These materials are hysteretic. Because of strong dipole-dipole interactions in ferromagnetic materials, the models available for these materials are complex and difficult to use. The energy of thermal fluctuations depends linearly on temperature. For this reason if a ferromagnetic material is heated, in a certain temperature it becomes paramagnetic. This transition temperature is called the Curie temperature T_c. Curie temperature is fairly high for most of the ferromagnetic materials. For iron $T_c = 1043\text{K}$.

When a ferromagnetic material is heated beyond T_c, it becomes paramagnetic, and during this heating process, the entropy of the materials is increased. In the following lemmas, this fact is used together with entropy relations for a paramagnetic material to show an upper bound for the entropy in a ferromagnetic material. The first lemma is used to show that the Helmholtz free energy has a lower bound.

LEMMA 7. *For a paramagnetic material at a constant temperature, the entropy S has an upper bound.*

Proof. The strength of a magnetic dipole is denoted by a constant positive half-integer J. This constant depends on the material under discussion. The following equations define entropy for a single dipole in a paramagnetic sample [13, pp. 213, 215, and 259]:

$$\begin{aligned} \beta &= \frac{1}{kT}, \\ \eta &= c\beta \left\| H \right\|, \\ Z &= \frac{\sinh\left[(J+\frac{1}{2})\eta\right]}{\sinh\left[\frac{1}{2}\eta\right]}, \\ S &= k\left(\ln Z - \beta \frac{\partial \ln Z}{\partial \beta}\right), \end{aligned} \tag{20}$$

where c is a positive constant and k is the Boltzmann constant $k = 1.38e-23\frac{\text{J}}{\text{K}}$.

In a paramagnetic sample with N dipoles, total magnetic entropy is simply N times the entropy of a single dipole. Total magnetic entropy is maximized when $H = 0$. (See the appendix.) This result is consistent with physics since in the presence of an external magnetic field, dipoles become oriented and the overall system disorder is reduced. Thus,

$$S_{\max} = S_{H=0} = kN \ln(2J+1). \tag{21}$$

Thus, at a constant temperature, the magnetic portion of entropy has an upper bound, $S_{\max}$.

The nonmagnetic portion of the entropy is a function of temperature and external load. At any temperature, this entropy is maximized for the highest possible (tensile) external load. This means that at any temperature, the nonmagnetic portion of the entropy has an upper bound. Thus at any temperature, the total entropy has an upper bound. □

The paramagnetic state is usually obtained at a high temperature. In order to have an upper bound for entropy in normal working conditions of the material, the lemma above should be extended to ferromagnetic materials.

LEMMA 8. *For any magnetic material at a constant temperature, the entropy S has an upper bound.*

Proof. Lemma 7 states that the entropy has an upper bound for the paramagnetic state. Here we are interested in the ferromagnetic state.

To obtain a relation for entropy in the ferromagnetic state, consider a process in which the ferromagnetic material is heated from an arbitrary initial state to a state in which the material is paramagnetic. The entropy and temperature for the initial state are S_i and T_i, respectively. For the paramagnetic state, the entropy and temperature are S_p and T_p, respectively. From Lemma 7, it is known that S_p has an upper bound.

The entropy is a function of the system states [12, p. 217]. The difference between any two arbitrary states is only a function of the states. This difference is independent of the process which connects the two states. This fact holds for the process mentioned above. The difference $S_p - S_i$ does not depend on the process as long as the initial and final conditions remain the same. For simplicity, consider a process in which the temperature is increased monotonically.

Since the temperature is always increasing during this process, there should be a nonnegative heat flow to the material during the process:

$$\frac{dQ}{dt} \geq 0. \tag{22}$$

The inequality of Clausius states that for any process $\frac{dS}{dt} \geq \frac{1}{T}\frac{dQ}{dt}$. As a result, in this process $\frac{dS}{dt} \geq 0$ or $S_p - S_i \geq 0$. Since S_p has an upper bound, S_i is bounded from above. This concludes the proof. ∎

The following is an immediate result of the lemma above.

THEOREM 9. *For a constant temperature, the Helmholtz free energy $\psi = U - TS$ is bounded from below.*

Proof. Lemma 8 states that the entropy has an upper bound. This means that $-TS$ has a lower bound. The internal energy U has a lower bound. This results in ψ being bounded from below. ∎

THEOREM 10. *The following passivity condition is satisfied when the storage function is the Helmholtz free energy ψ:*

$$\psi_i + \int_{t_i}^{t_f} \left\langle F, \frac{dX}{dt} \right\rangle dt \geq \psi_f. \tag{23}$$

Here, subscripts i and f denote initial and final conditions, respectively; F is the generalized force applied to the system; $\binom{\mu_0 H}{\sigma}$, X is the generalized system output; $\binom{M}{\varepsilon}$; and σ and ε are stress and strain, respectively.

Proof. If the temperature is constant, (18) can be written as

$$\frac{d\psi}{dt} \leq \frac{dW}{dt}, \tag{24}$$

where $\frac{dW}{dt} = \left\langle F, \frac{dX}{dt} \right\rangle$ is the rate of magnetic/mechanical work done on the system.

If both sides are integrated from t_i to t_f, we obtain

$$\psi_f - \psi_i \leq \int_{t_i}^{t_f} \left\langle F, \frac{dX}{dt} \right\rangle dt \tag{25}$$

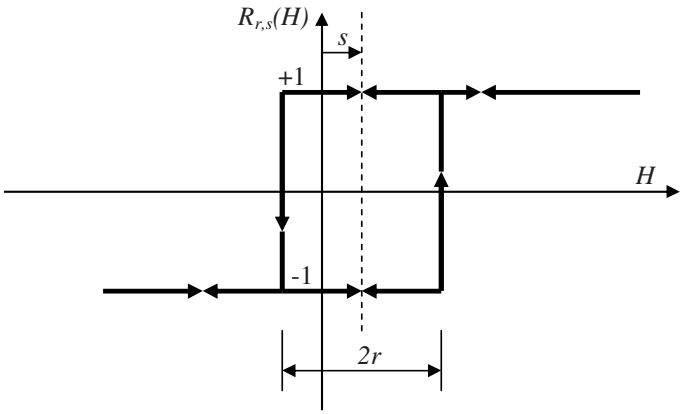

FIG. 3. *The Preisach relay.*

or

$$\psi_i + \int_{t_i}^{t_f} \left\langle F, \frac{dX}{dt} \right\rangle dt \geq \psi_f. \tag{26}$$

Theorem 9 shows that the Helmholtz free energy is bounded from below, which means that it is a valid storage function. This concludes the proof. □

The proof above shows the passivity of a magnetostrictive system with a three-dimensional magnetic field and a one-dimensional stress-strain. In this proof, no model for the magnetostrictive material is assumed. Passivity is shown with fundamental laws of physics only. In fact, the theorem above can be applied to any model for magnetostrictive materials.

4. The Preisach model. The Preisach model [2] is a very common model in the smart materials literature; for examples, see [3, 4, 14, 15]. In [15], it is used to model magnetostrictive materials. It has been shown that this model can represent magnetostrictive materials accurately [16]. This model is briefly explained here; for a detailed description, see [2]. In this model, a one-dimensional magnetic field is assumed, which results in the magnetic field H and magnetization M being scalars. It is assumed that the output is the weighted sum of the output of a continuum of hysteresis relays. The output of each relay can be either $+1$ or -1, determined by the previous relay value and the input, magnetic field H. In Figure 3 a typical hysteresis relay is shown.

The model output is

$$M(t) = \int_0^\infty \int_{-\infty}^\infty R_{r,s}[H(\cdot)](t)\mu(r,s)dsdr. \tag{27}$$

Here, $R_{r,s}$ is the output of the relay defined by r and s, and $\mu(r,s)$ is a weight function determined by experimental data.

Consider a two-dimensional coordinate system with variables r and s as shown in Figure 4. Each point r, s in this coordinate system is in a one-to-one relation with a Preisach relay $R_{r,s}$ and its corresponding weight $\mu(r,s)$. The plane defined by variables r and s is called the Preisach plane. Because the system input is limited, the relays with a large r or s do not change and cannot contribute to a change in the

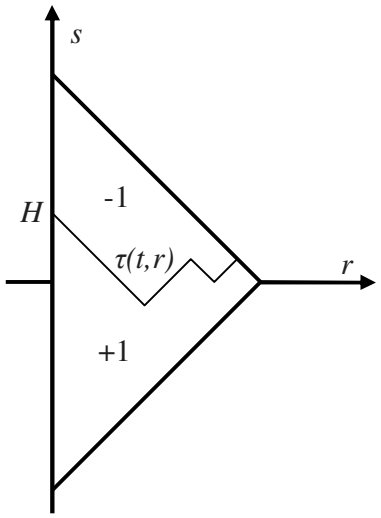

FIG. 4. *A typical Preisach plane boundary.*

model output. For simplicity, it is assumed that the weight function $\mu(r,s)$ is zero for these relays. In Figure 4, these relays are outside of the bold triangle. Since the output is not affected by these relays, they are not considered.

If, in the Preisach plane, the relays equal to -1 are separated with a line from the relays at $+1$, a boundary $s = \tau(t,r)$ will be produced, as shown in Figure 4. This boundary is important since if $\tau(t,r)$ is available, the output of all relays are known. Thus, knowledge of $\tau(t,r)$ and the input $H(t)$ determines future values of $\tau(t,r)$ and hence $M(t)$. In other words, $\tau(t,r)$ contains the *memory* of the system. The Preisach model is a dynamical system with $\tau(t,r)$ as the state [17]. The model output can be rewritten in terms of the boundary:

$$M(t) = 2\int_0^\infty \int_{-\infty}^{\tau(t,r)} \mu(r,s)dsdr - \int_0^\infty \int_{-\infty}^{\infty} \mu(r,s)dsdr. \tag{28}$$

Note that the Preisach boundary $\tau(t,r)$ and the vertical axis $r = 0$ in Figure 4 intersect at the current input value; that is,

$$\tau(t,0) = H. \tag{29}$$

4.1. Energy-based Preisach model. In this model, a physical model for magnetostrictive materials is used to develop a special type of Preisach model that is based on energy considerations [7, 16]. Here, the material is assumed to be composed of a large number of weakly interacting dipoles. The Helmholtz free energy for a single dipole can be modeled by three parabolas [7], [15, p. 188] (Figure 5):

$$\psi(M,\varepsilon) = \frac{1}{2}Y\varepsilon^2 - Y\gamma\varepsilon M^2 + \begin{cases} \frac{\mu_0\eta'}{2}(M+M_R)^2, & M \le -M_I, \\ \frac{\mu_0\eta'}{2}(M-M_R)^2, & M \ge M_I, \\ \frac{\mu_0\eta'}{2}(M_R-M_I)(M_R-\frac{M^2}{M_I}), & |M| < M_I, \end{cases} \tag{30}$$

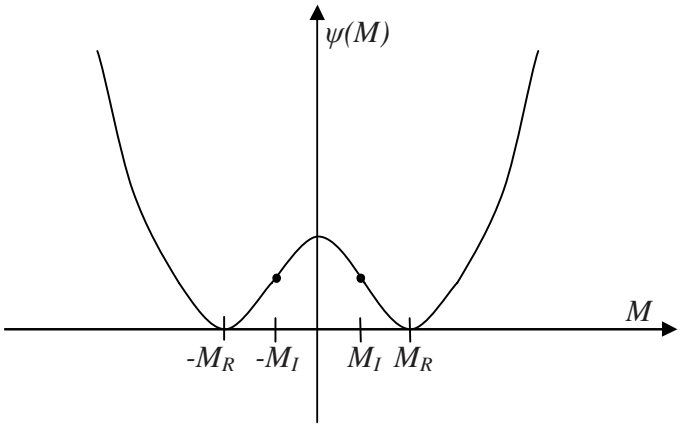

FIG. 5. *The Helmholtz free energy.*

where the variable M is the magnetization for the dipole, the parameter η' is a constant, γ is the magnetomechanical coupling constant, and Y is Young's modulus. The parameter M_R is the remanence magnetization. In the absence of strain ε, $\pm M_R$ are the minima of ψ. The parameter M_R is assumed to be the same for all dipoles. The parameter M_I is the inflection point where the second derivative of ψ changes sign. Unlike M_R, because of the nonhomogeneities and imperfections in the material, M_I is different for each dipole. For a valid Helmholtz free energy $M_R > M_I$. This ensures that the Helmholtz free energy has two distinct minima, as shown in Figure 5.

Define H_0 to be the local magnetic field at a dipole. Because of the imperfections and nonhomogeneities in the material, the local magnetic field H_0 might not be equal to the external magnetic field H. It is assumed that the difference $s = H - H_0$ is constant over time for each dipole.

The parameters s and M_I describe each dipole. Define

$$r = \eta'(M_R - M_I) + \frac{2}{\mu_0} Y\gamma\varepsilon M_I. \tag{31}$$

It will be shown later that it is easier to use r as defined in (31) to describe each dipole instead of M_I. This definition of r is consistent with r for a Preisach relay, as shown in Figure 3.

For a dipole, the Gibbs energy is

$$G_{r,s}(H_0, M_{r,s}, \sigma, \varepsilon) = \psi_{r,s}(M_{r,s}, \varepsilon) - \mu_0 H_0 M_{r,s} - \sigma\varepsilon, \tag{32}$$

as shown in Figure 6.

Consider a single dipole in a process in which the temperature, magnetic field H, and stress are constant. In this case, (16) simplifies to

$$\frac{dG_{r,s}}{dt} \leq 0. \tag{33}$$

This relation states that during this process, G has to either stay constant or decrease. At a stable equilibrium point, the Gibbs energy is minimized [15, pp. 65 and 184]. In this case, the derivative of Gibbs energy has to be zero with respect to

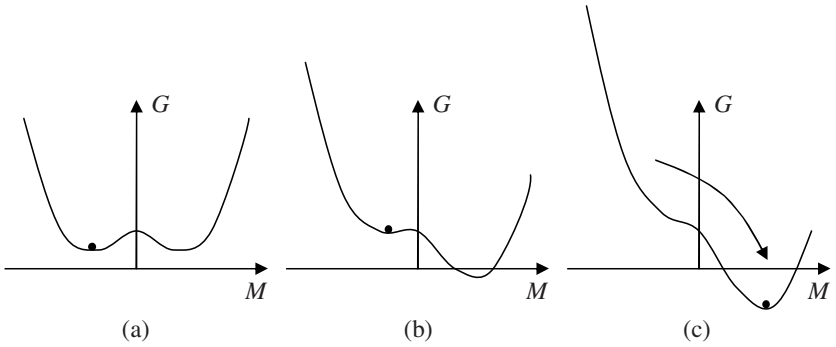

FIG. 6. (a) *Gibbs energy when* $H_0 = 0$, (b) *Gibbs energy for a positive* H_0, (c) *if* H_0 *is further increased, at some point, only one minimum exists.*

unconstrained variables:

$$\left(\frac{\partial G_{r,s}(H_0, M_{r,s}, \sigma, \varepsilon)}{\partial M_{r,s}}\right)_{T,H_0,\sigma,\varepsilon} = 0, \tag{34}$$

$$\left(\frac{\partial G_{r,s}(H_0, M_{r,s}, \sigma, \varepsilon)}{\partial \varepsilon}\right)_{T,H_0,\sigma,M_{r,s}} = 0. \tag{35}$$

By combining (32) and (34), the following relation is obtained:

$$\mu_0 H_0 = \left(\frac{\partial \psi}{\partial M_{r,s}}\right)_{T,\varepsilon}. \tag{36}$$

In a magnetic system with many dipoles, the dipole dynamics are very fast. If the magnetic field is not very rapidly changing, the magnetic field appears to be almost constant for each dipole over the time constant of the dipole. The magnetization for a dipole is a minimum of the Gibbs energy.

By combining (30), (32), and (36), the equilibrium magnetization for a dipole is obtained:

$$M^*_{r,s} = \frac{H - s + R_{r,s}\eta' M_R}{\eta' - \frac{2Y\gamma\varepsilon}{\mu_0}}. \tag{37}$$

If the dipole is in the left minimum in Figure 6(a), $R_{r,s} = -1$, and if the dipole is in the right minimum, $R_{r,s} = +1$.

As seen in Figure 6, if $H_0 = 0$, two minima exist. For a small positive H_0 as shown in Figure 6(b), still two minima exist, but if H_0 is further increased, at some point, one disappears, as shown in Figure 6(c). At this time, dipole magnetization moves to the new minimum. This transition is shown with an arrow in Figure 6(c).

Using (31), it can be shown that if $H \geq s + r$, the $R = -1$ minimum does not exist. Similarly, for $H \leq s - r$, the $R = +1$ minimum vanishes. For $s - r < H < s + r$, two minimums exist, which means that both $R = -1$ and $R = +1$ are possible. It is seen that for the Preisach relay introduced in Figure 3, the output -1 is nonexistent if $H \geq s + r$, and $+1$ vanishes if $H \leq s - r$. For the values between $s - r$ and $s + r$, both outputs are possible. This similarity between the dipole and a Preisach relay shows that the definition of r and s are consistent with r and s of a Preisach relay.

For a large magnetic field, the dipole magnetization is the right minimum. At this minimum, the Gibbs energy is

$$G_{r,s} = \frac{1}{2}Y\varepsilon^2 + \frac{\mu_0}{2}\eta M_R^2 - \frac{\mu_0(H - s + \eta M_R)^2}{2(\eta - \frac{2Y\gamma\varepsilon}{\mu_0})} - \sigma\varepsilon. \tag{38}$$

It is seen that, if $\varepsilon = 0$, the Gibbs energy can be made arbitrarily small by increasing H. This means that the Gibbs energy is unbounded from below.

Assuming a distribution $\mu(r, s)$ for the dipoles, the overall magnetization can be obtained:

$$M_{Tot} = C\int_0^\infty \int_{-\infty}^\infty M_{r,s}^* \mu(r, s) ds dr. \tag{39}$$

Define $\mathcal{I}_n$ to be

$$\mathcal{I}_n = \int_0^\infty \int_{-\infty}^\infty s^n \mu(r, s) ds dr, \tag{40}$$

where $n = 0, 1$, or 2. Using (37), M_{Tot} can be written as follows:

$$M_{Tot} = \frac{C}{\eta' - \frac{2Y\gamma\varepsilon}{\mu_0}} \left[\mathcal{I}_0(H - M_R\eta') - \mathcal{I}_1 + 2M_R\eta' \int_0^\infty \int_{-\infty}^{\tau(t,r)} \mu(r, s) ds dr\right], \tag{41}$$

where C is a constant and $\tau(t, r)$ is the Preisach boundary for the relay configuration $R_{r,s}$. The experimental data can be used to find the optimum weight function $\mu(r, s)$. A few common choices for $\mu(r, s)$ can be found in [16, 18].

Unlike the Preisach model, magnetization in this model depends on ε. In this model, σ and H are the inputs. The Preisach plane boundary $\tau(t, r)$ and ε are the system states. The outputs are ε and M. The magnetization is determined by (41). Combining (30), (32), and (35), we obtain

$$\varepsilon = \frac{\sigma}{Y} + \gamma M^2, \tag{42}$$

which determines strain ε.

4.2. Helmholtz free energy using the Preisach model. In this section, the total Helmholtz free energy for a magnetostrictive material is calculated using the physical Preisach model. Since this function is the system storage function, it is written as a function of system states $\tau(t, r)$ and ε.

As stated before, the local magnetic field H_0 might not be equal to the external magnetic field H. This difference between H and H_0 should have some effect on the energy functions. For example, consider a dipole with a negative s when the dipole magnetization is increased by dM and the external magnetic field H is constant: Work done by the external magnetic source is HdM, and work done on the dipole is $H_0dM = HdM - sdM$. It is seen that the work done on the dipole is more that the work done by the external magnetic field. This extra work is not done by the external field. The imperfections and nonhomogeneities which are the source of the difference between H and H_0 should have done this work on the dipole. As a result, they need to be considered when the overall system Helmholtz free energy is computed.

From (32), we have $G_{r,s}(H_0, M_{r,s}) = \psi_{r,s}(M_{r,s}) - \mu_0 H_0 M_{r,s} - \sigma\varepsilon$. Define $\bar{\psi}_{r,s}(M_{r,s})$ and $\bar{G}_{r,s}(H, M_{r,s})$ to be the Helmholtz free energy and Gibbs energy, respectively, written in terms of external variables. When the system is viewed from an

external point of view, the combined effect of the dipole and the imperfections is seen. To find $\bar{\psi}_{r,s}(M_{r,s})$ and $\bar{G}_{r,s}(H, M_{r,s})$, an assumption for the imperfections and non-homogeneities must be made and, based on that, the contribution to the Helmholtz free energy computed. Another approach is to construct $\bar{\psi}_{r,s}(M_{r,s})$ by studying the equilibrium points of the system for a constant magnetic field.

The equilibrium points for a constant magnetic field in terms of the external variables (H,$M_{r,s}$) can be obtained via two methods:

1. The equilibrium condition can be written for $\bar{G}_{r,s}(H, M_{r,s})$.
2. The system parameters can be transformed to the local variables (H_0, $M_{r,s}$). The equilibrium condition is written for $G_{r,s}(H_0, M_{r,s})$, and the results are transformed back to the external variables.

These two methods must be equivalent.

The equilibrium conditions for $\bar{G}_{r,s}(H, M_{r,s})$ and $G_{r,s}(H_0, M_{r,s})$ are

$$\left(\frac{\partial \bar{G}_{r,s}(H, M_{r,s})}{\partial M_{r,s}}\right)_{T,H} = 0, \qquad \left(\frac{\partial G_{r,s}(H_0, M_{r,s})}{\partial M_{r,s}}\right)_{T,H_0} = 0, \tag{43}$$

where $H = H_0 + s$ and s is assumed constant. Further,

$$\begin{aligned}\left(\frac{\partial G(H_0, M_{r,s})}{\partial M_{r,s}}\right)_{T,H_0} &= \left(\frac{\partial}{\partial M_{r,s}}\right)_{T,H} (\psi(M_{r,s}) - \mu_0 H_0 M_{r,s} - \sigma\varepsilon) \\ &= \left(\frac{\partial}{\partial M_{r,s}}\right)_{T,H} (\psi(M_{r,s}) - \mu_0 H M_{r,s} + \mu_0 s M_{r,s} - \sigma\varepsilon) \\ &= 0.\end{aligned} \tag{44}$$

Now, $\bar{G}_{r,s}(H, M_{r,s})$ equals $G_{r,s}(H - s, M_{r,s})$ or

$$\bar{G}_{r,s}(H, M_{r,s}) = \psi(M_{r,s}) - \mu_0 H M_{r,s} + \mu_0 s M_{r,s} - \sigma\varepsilon. \tag{45}$$

It can be shown that the equilibrium conditions (43) are identical. Defining $\bar{\psi}_{r,s}(M_{r,s})$ so that $\bar{G}_{r,s}(H, M_{r,s}) = \bar{\psi}_{r,s} - \mu_0 H M_{r,s} - \sigma\varepsilon$, analogously with (32), we have

$$\bar{\psi}_{r,s}(M_{r,s}) = \psi(M_{r,s}) + \mu_0 s M_{r,s}. \tag{46}$$

Equation (37) gives the equilibrium magnetization for a dipole. By combining (30), (37), and (46), the equilibrium value of $\bar{\psi}_{r,s}$ for each dipole is obtained:

$$\bar{\psi}^*_{r,s} = \frac{1}{2} Y \varepsilon^2 + \frac{\frac{\mu_0}{2}(H^2 - s^2) - \eta' M_R (Y\gamma\varepsilon M_R - \mu_0 s R_{r,s})}{\eta' - \frac{2Y\gamma\varepsilon}{\mu_0}}. \tag{47}$$

Similar to (39), by assuming a distribution for r and s, the Helmholtz free energy for the entire system can be found using the superposition principle:

$$\psi_{Tot}(\tau(t,r), \varepsilon) = C \int_0^\infty \int_{-\infty}^\infty \bar{\psi}^*_{r,s} \mu(r,s) ds dr. \tag{48}$$

By combining (29), (47), and (48), the following equation is obtained:

$$\begin{aligned}\psi_{Tot}(\tau(t,r), \varepsilon) = \frac{C\mathcal{I}_0}{2} Y\varepsilon^2 + \frac{C}{\eta' - \frac{2Y\gamma\varepsilon}{\mu_0}} \Bigg(&\frac{\mu_0 \mathcal{I}_0 \tau^2(t,0)}{2} - \eta' Y \gamma \varepsilon M_R^2 \mathcal{I}_0 \\ &+ \eta' M_R \mu_0 A - \frac{\mu_0}{2} \mathcal{I}_2 \Bigg),\end{aligned} \tag{49}$$

where $A = \int_0^\infty \int_{-\infty}^\infty R_{r,s} s \mu(r,s) ds dr = 2 \int_0^\infty \int_{-\infty}^{\tau(t,r)} s \mu(r,s) ds dr - \mathcal{I}_1$.

This is the value of the Helmholtz free energy, the storage function for the magnetostrictive system, for any ε and Preisach boundary $\tau(t,r)$. The only nontrivial aspect of calculating $\psi_{Tot}(\tau(t,r),\varepsilon)$ is efficient computation of A. It is seen that the double integral of A is very similar to the double integral used for computing M (27). In fact, any efficient algorithm used for the computation of M can be used here, for example that on [2, p. 37]; only the weight function is slightly different.

4.3. Minimum of the storage function. In this section, the Preisach boundary that globally minimizes the storage function is obtained.

Suppose that when $\tau(t,r)=\tau^*(t,r)$ and $\varepsilon=\varepsilon^*$, $\psi_{Tot}(\tau(t,r),\varepsilon)$ is globally minimized. If ε is held fixed at $\varepsilon=\varepsilon^*$ and $\tau(t,r)$ is changed, $\psi_{Tot}(\tau(t,r),\varepsilon^*)$ is minimized when $\tau(t,r)=\tau^*(t,r)$. This means that $\tau^*(t,r)$ globally minimizes the following function:

$$\psi_{Tot}(\tau(t,r),\varepsilon^*)=\frac{C\mathcal{I}_0}{2}Y\varepsilon^{*2}+\frac{C}{\eta'-\frac{2Y\gamma\varepsilon^*}{\mu_0}}\left(\frac{\mu_0\mathcal{I}_0\tau^2(t,0)}{2}-\eta' Y\gamma\varepsilon^* M_R^2\mathcal{I}_0 + \eta' M_R\mu_0 A-\frac{\mu_0}{2}\mathcal{I}_2\right). \tag{50}$$

The following terms are the only variable parts of the storage function:

$$\begin{aligned} F_1(\tau(t,r)) &= \frac{\mu_0\mathcal{I}_0\tau^2(t,0)}{2},\\ F_2(\tau(t,r)) &= A. \end{aligned} \tag{51}$$

Assume that the weight function $\mu(r,s)$ is nonnegative for all r and s. Since $\eta'-\frac{2Y\gamma\varepsilon^*}{\mu_0}$ is a positive quantity, if F_1 and F_2 are minimized at the same time, the storage function is minimized. Function F_1 is minimized when $\tau(t,0)=0$. Function F_2 is minimized when A is minimized:

$$A=2\int_0^\infty\int_{-\infty}^{\tau(t,r)} s\mu(r,s)dsdr-\mathcal{I}_1. \tag{52}$$

The sign of the integrand equals the sign of s. This integration is minimized when the region of integration is the subset of the Preisach plane on which the integrand is negative. This is the lower half of the Preisach plane. Thus, the integration is minimized when the boundary $\tau(t,r)=0$. This Preisach plane boundary is shown in Figure 7.

Function F_2 is globally minimized with the boundary $\tau(t,r)=0$. Since for this boundary $\tau(t,0)=0$, this boundary also globally minimizes F_1. This results in global minimization of the storage function.

It is commonly seen that the weight function $\mu(r,s)$ is an even function of s; that is, $\mu(r,s)=\mu(r,-s)$ for all r and s [16, 18]. If this condition holds, by substituting the Preisach boundary $\tau(t,r)=0$ into (41), it is seen that the resulting magnetization is zero. In this case there is no magnetic field H, magnetization M, or flux density B. This state is called the demagnetized state and is the state of lowest "energy" for the system.

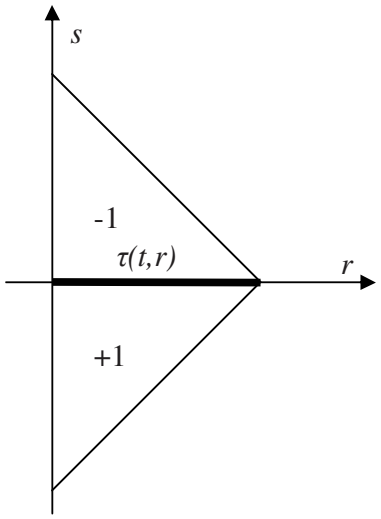

FIG. 7. *The global minimum Preisach boundary.*

5. Storage function in the presence of a constant input. When the stress and magnetic field applied to the system include a constant portion, the system can be simplified by redefining the input as $\bar{u} = \binom{\mu_0(H - H_{const})}{\sigma - \sigma_{const}}$, while the output is not changed. In this case, the system is passive with the following storage function:

$$\psi^F = \psi_{Tot} - \mu_0 \langle H_{const}, M_{Tot} \rangle - \sigma_{const}\varepsilon, \tag{53}$$

where ψ_{Tot} is the system Helmholtz free energy and M_{Tot} is the system magnetization. This situation is analogous to the example of a spring with a constant imposed force, such as gravity, discussed in section 2.

THEOREM 11. *In the presence of a constant input, the following passivity condition is satisfied when the storage function is* ψ^F*:*

$$\psi_i^F + \int_{t_i}^{t_f} \left\langle \bar{u}, \frac{dX}{dt} \right\rangle dt \geq \psi_f^F. \tag{54}$$

Subscripts i *and* f *denote initial and final conditions, respectively, and* $X = \binom{M}{\varepsilon}$ *is the generalized displacement.*

Proof. If the definition of $\bar{u}$ and ψ^F is substituted into the result of Theorem 10, the result is

$$\begin{aligned} \psi_i^F + \mu_0 \langle H_{const}, M_{Tot,i} \rangle + \sigma_{const}\varepsilon_i + \int_{t_i}^{t_f} \left\langle \bar{u} + \begin{pmatrix} \mu_0 H_{const} \\ \sigma_{const} \end{pmatrix}, \frac{dX}{dt} \right\rangle dt \\ \geq \psi_f^F + \mu_0 \langle H_{const}, M_{Tot,f} \rangle + \sigma_{const}\varepsilon_f. \end{aligned} \tag{55}$$

This simplifies to

$$\psi_i^F + \int_{t_i}^{t_f} \left\langle \bar{u}, \frac{dX}{dt} \right\rangle dt \geq \psi_f^F. \tag{56}$$

Since both M_{Tot} and ε have a lower bound and an upper bound, existence of a lower bound for ψ_{Tot} implies that ψ^F has a lower bound. The proof is complete. ∎

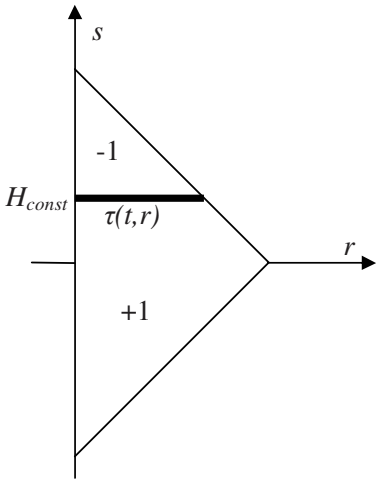

FIG. 8. *The global minimum Preisach boundary in the presence of a constant input.*

The storage function ψ^F can be written as a function of the Preisach boundary $\tau(t,r)$ and ε by combining (49) and (53):

$$\psi^F(\tau(t,r),\varepsilon) = \frac{C\mathcal{I}_0}{2}Y\varepsilon^2 + \frac{C}{\eta' - \frac{2Y\gamma\varepsilon}{\mu_0}}\Bigg(\frac{\mu_0\mathcal{I}_0(\tau(t,0)-H_{const})^2}{2} - \frac{\mu_0\mathcal{I}_0H_{const}^2}{2} - \eta' Y\gamma\varepsilon M_R^2\mathcal{I}_0 + \mu_0 H_{const}M_R\eta'\mathcal{I}_0 + \mu_0 H_{const}\mathcal{I}_1 - \eta' M_R\mu_0\mathcal{I}_1 + \mu_0\eta' M_R\bar{A} - \frac{\mu_0}{2}\mathcal{I}_2\Bigg) - \sigma_{const}\varepsilon, \tag{57}$$

where $\bar{A} = 2\int_0^\infty \int_{-\infty}^{\tau(t,r)}(s - H_{const})\mu(r,s)dsdr$. Using an argument similar to that of the previous section, it can be shown that the following boundary minimizes the storage function:

$$\tau(t,r) = H_{const}. \tag{58}$$

This boundary is shown in Figure 8. For a constant input, this is the state of minimum energy. The magnetization in this state is the anhysteretic magnetization.

6. Conclusions. In this article, magnetostrictive transducers were introduced in a dynamical system framework. Passivity of this system was shown using fundamental physical relations. For the energy-based Preisach model, the system states were defined, and the storage function was computed. System equilibrium points were also identified and discussed.

The passivity results discussed in this paper can be used to show the stability of a closed-loop system. Future work includes the design and optimization of a robustly stabilizing controller for magnetostrictive transducers.

Appendix. The maximization of entropy. In this appendix, it is shown that the entropy function (20) is maximized when $H = 0$.

From subsection 3.4, the following relations define entropy for a paramagnetic sample with N dipoles:

$$\begin{aligned} Z &= \frac{\sinh\left[(J+\frac{1}{2})\eta\right]}{\sinh\left[\frac{1}{2}\eta\right]}, \\ S &= kN\left(\ln Z - \beta\frac{\partial \ln Z}{\partial \beta}\right), \\ \eta &= c\beta\left\|H\right\|, \\ \beta &= \frac{1}{kT}, \end{aligned} \tag{59}$$

where c is a positive constant.

Define $D = \frac{\eta}{2} = \frac{c\beta}{2}\left\|H\right\|$ and $q = 2J+1$. Since J is a positive half-integer, q is an integer greater than one. We can write

$$S = kN\left(\ln\frac{\sinh qD}{\sinh D} - qD\coth qD + D\coth D\right). \tag{60}$$

This function is not defined at $D=0$, but $\lim_{D\to 0} S(D)$ exists:

$$\begin{aligned} \lim_{D\to 0} S(D) &= \lim_{D\longrightarrow 0} kN\left(\ln\frac{\sinh qD}{\sinh D} + \frac{D\cosh D\sinh qD - qD\cosh qD\sinh D}{\sinh D\sinh qD}\right) \\ &= \lim_{D\longrightarrow 0} kN\left(\ln\frac{qD+h.o.t.}{D+h.o.t.} + \frac{\frac{D^4}{6}(2q-2q^3)+h.o.t.}{qD^2+h.o.t.}\right) \\ &= kN\ln q. \end{aligned} \tag{61}$$

For $D\neq 0$, $S(D) = S(-D)$; i.e., this is an even function. We do not need to analyze this function for both positive and negative values of D. For simplicity $D>0$ is studied.

If $D>0$,

$$\frac{dS}{dD} = kN\left(\frac{q^2 D}{\sinh^2 qD} - \frac{D}{\sinh^2 D}\right). \tag{62}$$

It will be shown that for $D>0$, $\frac{dS}{dD} < 0$. Consider the Taylor series of the following expression:

$$\begin{aligned} \sinh qD - q\sinh D &= qD + \frac{q^3D^3}{3!} + \frac{q^5D^5}{5!} + \cdots - qD - \frac{qD^3}{3!} - \frac{qD^5}{5!} - \cdots \\ &= q\left((q^2-1)\frac{D^3}{3!} + (q^4-1)\frac{D^5}{5!} + \cdots\right). \end{aligned} \tag{63}$$

Since $q>1$ and $D>0$ all of the terms in the Taylor series are positive. It follows that

$$\sinh qD - q\sinh D > 0. \tag{64}$$

This inequality can be written as

$$1 < \frac{\sinh qD}{q\sinh D} \tag{65}$$

or

$$1 < \frac{\sinh^2 qD}{q^2 \sinh^2 D}. \tag{66}$$

This inequality can be further written as

$$\frac{q^2}{\sinh^2 qD} - \frac{1}{\sinh^2 D} < 0. \tag{67}$$

This terms appears in the derivative of S with respect to D:

$$\frac{dS}{dD} = kND\left(\frac{q^2}{\sinh^2 qD} - \frac{1}{\sinh^2 D}\right). \tag{68}$$

Since $D > 0$, this implies that $\frac{dS}{dD} < 0$.

Since $\frac{dS}{dD} < 0$, S can be increased by lowering D, or

$$\sup_{D>0} S(D) = \lim_{D\to 0} S(D) = kN \ln q. \tag{69}$$

Since $S(D)$ is an even function, this result can be extended to all values of $D \neq 0$: $kN \ln q$ is an upper bound for $S(D)$.

REFERENCES

[1] J. C. Willems, *Dissipative dynamical systems, Part* I: *General theory*, Arch. Ration. Mech. Anal., 45 (1972), pp. 321–351.

[2] I. Mayergoyz, *Mathematical Models of Hysteresis and Their Applications*, Elsevier Academic Press, Amsterdam, Boston, 2003.

[3] R. B. Gorbet, K. A. Morris, and D. W. L. Wang, *Passivity-based stability and control of hysteresis in smart actuators*, IEEE Trans. Control Systems Tech., 9 (2001), pp. 5–16.

[4] D. Hughes and J. T. Wen, *Preisach modeling of piezoceramic and shape memory alloy hysteresis*, Smart Materials and Structures, 6 (1997), pp. 287–300.

[5] X. Tan, J. S. Baras, and P. S. Krishnaprasad, *A dynamic model for magnetostrictive hysteresis*, Proceedings of the American Control Conference, 2 (2003), pp. 1074–1079.

[6] X. Tan and J. S. Baras, *Modeling and control of hysteresis in magnetostrictive actuators*, Automatica, 40 (2004), pp. 1469–1480.

[7] R. C. Smith, M. J. Dapino, and S. Seelecke, *Free energy model for hysteresis in magnetostrictive transducers*, J. Appl. Phys., 93 (2003), pp. 458–466.

[8] R. B. Gorbet and K. A. Morris, *Closed-loop position control of preisach hystereses*, J. Intell. Material Systems Struct., 14 (2003), pp. 483–495.

[9] W. M. Haddad, V. Chellaboina, and J. Oh, *Linear controller analysis and design for systems with input hystereses nonlinearities*, J. Franklin Inst., 340 (2003), pp. 371–390.

[10] M. Vidyasagar, *Nonlinear Systems Analysis*, Prentice–Hall, Englewood Cliffs, NJ, 1993.

[11] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input-output Properties*, Academic Press, New York, 1975.

[12] M. W. Zemansky and R. H. Dittman, *Heat and Thermodynamics*, McGraw–Hill, New York, Montreal, 1981.

[13] F. Reif, *Fundamentals of Statistical and Thermal Physics*, McGraw–Hill, New York, 1965.

[14] R. B. Gorbet, D. W. L. Wang, and K. A. Morris, *Preisach model identification of a two-wire SMA actuator*, Proceedings of the IEEE International Conference on Robotics and Automation, 3 (1998), pp. 2161–2167.

[15] R. C. Smith, *Smart Material Systems: Model Development*, Frontiers Appl. Math. 32, SIAM, Philadelphia, 2005.

[16] S. Valadkhan, K. A. Morris, and A. Khajepour, *A review and comparison of hysteresis models for magnetostrictive materials*, J. Intell. Material Systems Struct., to appear.

[17] R. B. Gorbet, K. A. Morris, and D. W. L. Wang, *Control of hysteretic systems: A state-space approach*, in Learning, Control, and Hybrid Systems, Y. Yamamoto, S. Hara, B. A. Francis, and M. Vidyasagar, eds., Springer-Verlag, London, 1999, pp. 432–451.

[18] R. C. Smith and M. J. Dapino, *A homogenized energy model for the direct magneto-mechanical effect*, Technical Report CRSC-TR05-31, Center for Research in Scientific Computation, North Carolina State University, Raleigh, NC, 2005, http://www.ncsu.edu/crsc/reports.htm.