Passivity-Based Stability and Control of Hysteresis in Smart Actuators

Robert B. Gorbet, Kirsten A. Morris, *Member, IEEE*, and David W. L. Wang, *Member, IEEE*

***Abstract*—The past decade has seen an increase in the use of smart materials in actuator design, notably for inclusion in active structures such as noise-reducing paneling or vibration-controlled buildings. Materials such as shape memory alloys (SMAs), piezoceramics, magnetostrictives and others all offer exciting new actuation possibilities. However, all of these materials present an interesting control challenge due to their nonlinear hysteretic behavior in some régimes. In this paper, we look at the energy properties of the Preisach hysteresis model, widely regarded as the most general hysteresis model available for the representation of classes of hysteretic systems. We consider the ideas of energy storage and minimum energy states of the Preisach model, and derive a passivity property of the model. Passivity is useful in controller design, and experimental results are included showing control of a differential shape memory alloy actuator using a passivity-based rate controller.**

***Index Terms*—Hysteresis, intelligent actuators, passivity, Preisach model, shape memory alloys (SMAs), stability, state-space models.**

I. Introduction

ACTUATORS made of so-called smart materials play an important role in the design and construction of smart structures. Specifically, composite materials with embedded piezoceramic, shape memory alloy, or magnetostrictive actuators have been proposed for use in smart structures capable of active noise, damage, and vibration control (e.g., [1]–[3]). Each of these actuator materials displays hysteretic behavior in some régime of operation. This nonlinearity can lead to instability in closed-loop operation (e.g., [4]), and complicates the task of controller design and analysis.

In order to develop controller design and analysis techniques applicable to a broad range of smart systems, we require a hysteresis model which can describe the behavior of each of these actuator materials. The Preisach model has been found suitable for representation of the hysteresis in shape memory alloys (SMAs) [5], piezoceramic [6], and magnetostrictive [7] actuators. However, little research has been done on Preisach model properties that pertain to controller design and system stability. This paper contributes to this area by providing an energy-based approach to controller design. As well, a state-space representation of the model is outlined which allows the application of a broader set of analysis and design tools than the classical input–output description.

The theoretical developments of this work were verified experimentally using a rotary actuator driven by two SMA wires in the differential configuration shown in Fig. 1. Counterclockwise motion is achieved by heating the top wire, by applying a voltage between the positive and ground terminals. Opposite motion occurs when the bottom wire is heated. Motion is controlled via a computer-commanded current amplifier, with two diodes interpreting the sign of the control signal and routing current to the appropriate wire. Full details on the actuator and its development can be found in [8].

The actuator displays a wide hysteresis loop as wire temperature is cycled, as shown in Fig. 2. In the figure, the sign of the temperature indicates which wire is being heated: the wire temperature is found from the magnitude of the indicated value. Results from [5] suggest that the behavior of this actuator can be described by the Preisach model. Active control of helicopter blade trim [9] and shape control of a hydrofoil [10] are two examples of smart structure applications of this configuration of antagonistic SMA actuators. Larger scale SMA actuators have also been proposed for active control of building vibration amplitude and resonant frequencies.

II. The Preisach Model

In the 1930s, Preisach developed an input–output model for magnetic hysteresis, based on assumptions on the behavior of elementary magnetic particles, known as dipoles, inside the magnetic material. In recent years, Preisach and Preisach-type models have been repeatedly and successfully applied to the modeling of smart material hysteresis (e.g., [5]–[7]). Mayergoyz has compiled one of the most comprehensive works on the Preisach model and its variations [11].

The basic building block of the Preisach model is the hysteresis relay γ. In this work, a relay is characterized by its half-width $r > 0$ and the input offset s, and is denoted by $\gamma_{r,s}$. The behavior of the relay is described schematically in Fig. 3, as is its place in the overall model structure. The model output is computed as the weighted sum of relay outputs; the value $\mu(r, s)$ represents the weighting of the relay $\gamma_{r,s}$. The relay output, and hence the Preisach model, is only defined for continuous inputs u. As this input varies with time, each individual relay adjusts its output according to the current input value, and the weighted sum of all relay outputs provides the overall system output (cf. Fig. 3)

$$y(t) = \int_0^\infty \int_{-\infty}^\infty \mu(r,s)\gamma_{r,s}[u](t)\,ds\,dr. \qquad (1)$$

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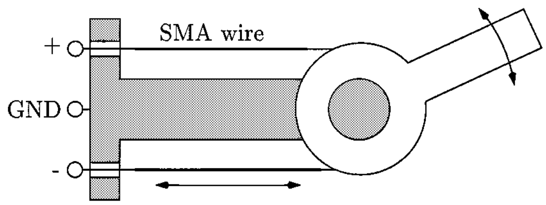

Fig. 1. Rotary actuator conceptual diagram.

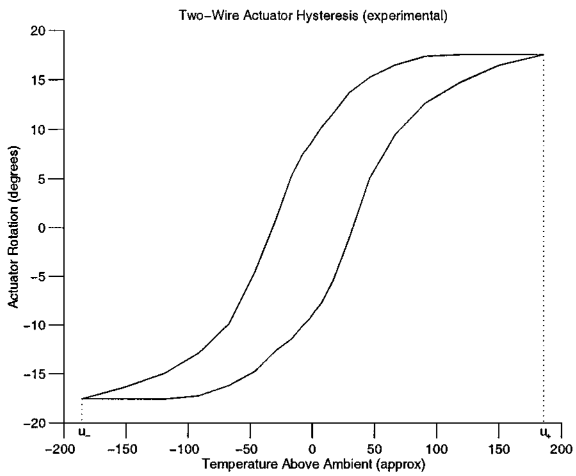

Fig. 2. Rotary actuator measured hysteresis.

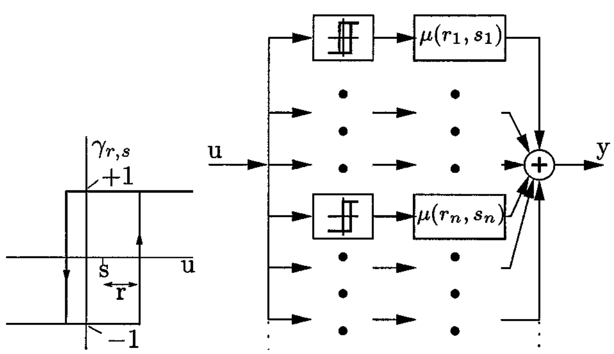

Fig. 3. Hysteresis relay and Preisach model structure.

The notation Γ will be used to denote the operator which maps a continuous input function to some output function according to (1): $y = \Gamma u$. $y_{r,s}$ represents the weighted output of the single relay $\gamma_{r,s}$: $y_{r,s}(t) = \mu(r,s)\gamma_{r,s}[u](t)$.

The identification of a relay $\gamma_{r,s}$ with the point (r,s) allows each relay to be uniquely represented as a point in $\mathbb{R}_+ \times \mathbb{R}$. This half-plane plays an important role in understanding the Preisach model, and is often referred to as the *Preisach plane,* $\mathcal{P}$. The collection of weights $\mu(r,s)$ forms the *Preisach weighting function* $\mu : \mathcal{P} \mapsto \mathbb{R}$. This weighting function is experimentally determined for a given system; see [11] and [12] for two different approaches to the identification problem.

In any physical setup, there are limitations which can be interpreted as a restriction on the support of μ. For instance, control input saturation, say at $\hat{u}$, means that some relays in $\mathcal{P}$ can never be exercised and cannot contribute to a change in output. This effectively restricts the domain of μ to a triangle in $\mathcal{P}$ defined by $\mathcal{P}_r = \{(r,s) \in \mathcal{P} \,|\, |s| \le \hat{u} - r\}$, illustrated in Fig. 4. In this

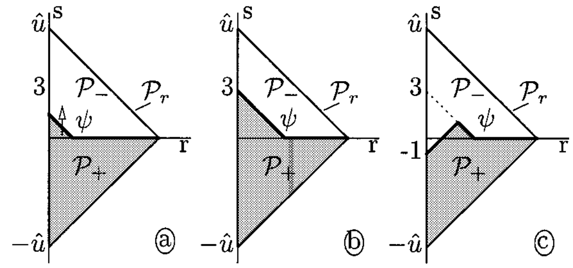

Fig. 4. Preisach boundary behavior.

case, μ effectively has compact support $\mathcal{P}_r$. We will henceforth assume that μ is only nonzero in some region $\mathcal{P}_r$. We will also assume that μ is bounded, piecewise continuous, and nonnegative inside $\mathcal{P}_r$; the set of such μ will be called $\mathcal{M}_p$. These are common assumptions when dealing with Preisach models for physical systems (e.g., [11] and [13]), and a model of the differential rotary SMA actuator identified in [5] satisfies $\mu \in \mathcal{M}_p$.

Assumption II.1: It will be assumed throughout that the Preisach weighting function μ belongs to $\mathcal{M}_p$. That is, that $\mu = 0$ outside the bounded set $\mathcal{P}_r$, and that μ is nonnegative, finite, and piecewise continuous inside $\mathcal{P}_r$.

In [14], it was shown that if μ is bounded and piece-wise continuous, then $\Gamma : C^0 \mapsto C^0$. If furthermore μ is nonnegative, then $\Gamma : W_1^2 \mapsto W_1^2$. The space W_1^2 is the linear space of real-valued functions satisfying $\int_{-\infty}^{\infty} (\dot{u}^2 + u^2)\, dt < \infty$.

One ambiguity remains in this definition of the model, and that is the question of the initial state of the relays γ. The output y depends not only on u but on the initial configuration of the relays of $\mathcal{P}_r$. It is common to assume an initial relay output of -1 if $s > 0$ and $+1$ otherwise (e.g., [13]). There is some physical justification for this choice. Since magnetic materials have weighting functions μ which are symmetric about $s = 0$, the model output corresponding to this assumed initial state is zero. It represents the *demagnetized* state of a magnetic material: the state in which no remnant magnetization is present [15].

The Preisach plane can be used to track individual relay states by observing the evolution of the *Preisach plane boundary*, ψ, in $\mathcal{P}_r$. First, the relays are divided into two time-varying sets, represented by the regions $\mathcal{P}_-$ and $\mathcal{P}_+$ defined as follows:

$$\mathcal{P}_\pm(t) = \{(r,s) \in \mathcal{P}_r \mid \text{ output of } \gamma_{r,s} \text{ at } t \text{ is } \pm 1\}. \quad (2)$$

It will become clear that each set is connected; ψ is the line separating $\mathcal{P}_+$ from $\mathcal{P}_-$. The boundary corresponding to the assumed initial relay configuration (the intersection of $s = 0$ and $\mathcal{P}_r$) will be denoted ψ^*.

As an example, suppose the input u starts at $u = 0$ and increases monotonically to $u = 3$. Initially, $\psi = \psi^*$. As u increases, relay outputs switch from -1 to $+1$ when $u = s + r$. Hence, $\mathcal{P}_+$ grows at the expense of $\mathcal{P}_-$, and the moving boundary defining this growth is the line $s = u - r$. In Fig. 4(a), the thick line represents ψ for some input value $0 < u < 3$; the arrow indicates that the sloped segment moves upwards as u increases. The line in Fig. 4(b) shows the state of $\mathcal{P}_r$ when $u = 3$. Similarly, if the input reverses direction at $u = 3$ and decreases monotonically to $u = -1$, relays in $\mathcal{P}_+$ switch over to $\mathcal{P}_-$ when $u = s - r$. A new segment is generated on ψ, corresponding to

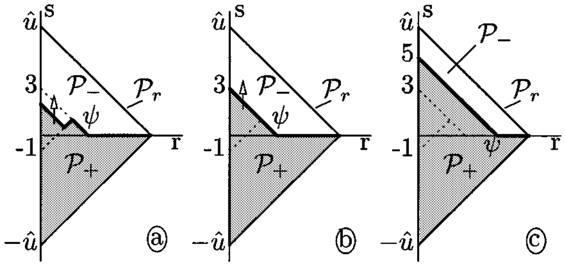

Fig. 5. Wiping out property.

the line $s = u + r$ [cf. Fig. 4(c)]. Subsequent input reversals generate further segments of ψ.

Note that the boundary ψ always intersects the axis $r = 0$ at the current input value. With ψ written as a map $\mathbb{R} \times \mathbb{R}_+ \mapsto \mathbb{R}$, then $\psi(t, 0) = u(t)$. If the boundary at time t is $\psi(t, r)$, applying an input for which $u(t) \neq \psi(t, 0)$ amounts to applying an input with a discontinuity at t. In this case, the output is not defined. This observation leads to the following definition.

Definition II.1 (Admissible Inputs): An input $u \in \mathcal{U}$ is said to be admissible to boundary ψ at time t if $u(t) = \psi(t, 0)$.

The Preisach plane boundary ψ is the memory of the Preisach model. When an arbitrary input is applied, monotonically increasing input segments generate boundary segments of slope -1, while monotonically decreasing input segments generate boundary segments of slope $+1$. Input reversals cause corners in the boundary. The history of past input reversals—and hence of hysteresis branching behavior—is "stored" in the corners of the boundary.

Some input extrema can remove the effects of previous extrema, essentially "wiping out" the memory of the model. This "wiping out" behavior is one of two necessary and sufficient conditions for existence of a Preisach model. For more on representation conditions, see [11]. The wiping out behavior is sketched in Fig. 5, and explained as follows. Consider once more the input of Fig. 4. Continuing from Fig. 4(c), suppose that u reverses and increases monotonically until it reaches $u = 5$. A segment of slope -1 sweeps upward through $\mathcal{P}_r$, switching relays from $\mathcal{P}_-$ to $\mathcal{P}_+$ [cf. Fig. 5(a)]. As the input reaches $u = 3$ and continues to increase, the two corners which had been generated by previous reversals at $u = 3$ and $u = -1$ merge and disappear [cf. Fig. 5(b)]. For $u > 3$, the influence of those two previous input extrema has been completely removed [cf. Fig. 5(c)]. So input extrema which exceed previous extrema in magnitude can "wipe out" part of the memory. Those extrema which remain in memory at any time form a *reduced memory sequence*, which will be discussed further in Section IV.

III. Preisach Model Input–Output Passivity

The concept of passivity and the related Passivity theorem are commonly used tools in the analysis and design of stabilizing controllers for robotic and other nonlinear systems. Passivity theory originates in the study of energy consumption and production by a system. Essentially, if there is no internal production of energy in a system, it is said to be passive. In physical systems, inputs and outputs are often "energy pairs," and their product gives instantaneous power consumption: force and velocity in mechanical systems, voltage and current in electrical systems. The observation that $u \cdot y$ represents consumed energy leads to the classical sector test for passivity. If, for any u, $\langle u, y\rangle \geq 0$—if the input–output graph lies within the first and third quadrants—then net energy consumption is positive for all inputs and the system is passive [16].

This input–output approach to passivity requires special care when dealing with hysteretic systems, since they have a multi-valued response in the region of hysteresis. For this reason, we introduce relations and accompanying relational definitions for the concepts commonly associated with passivity theory.

Definition III.1 (Relation): A *relation* H on $L_2[0, T]$ defines a set of ordered pairs $(u, y) \in L_2[0, T] \times L_2[0, T]$. The domain and range of H is defined as follows:

$$\mathrm{Do}(H) = \{u \in L_2[0, T] \mid \exists y \in L_2[0, T] \text{ s.t. } y = Hu\}$$
$$\mathrm{Ra}(H) = \{y \in L_2[0, T] \mid \exists u \in L_2[0, T] \text{ s.t. } y = Hu\}.$$

In general, a relation may be multivalued: for any $u \in \mathrm{Do}(H)$, there may be several $y \in L_2[0, T]$ such that $y = Hu$. It is this multivalued nature which makes relations useful in describing hysteretic systems. It is important to note that H need not be defined on the whole of $L_2[0, T]$.

In working with relations, we adopt the following convention: When something is said to hold (or is required to hold) for Hu, without qualification, it will be understood that it holds (or is required to hold) for all possible outputs corresponding to the input u. As a result of this convention, many of the relational system theory definitions closely resemble their standard counterparts, as in the following definition of finite gain. Note, however, that demonstrating finite gain of a relation is more involved, since (3) must hold not only for all u, but for every possible output for each of those inputs.

Definition III.2 (Finite Gain): A relation H is said to have *finite gain* if there exists $\gamma > 0$ such that, for all $T \geq 0$ and $u \in \mathrm{Do}(H)$

$$||Hu||_T \leq \gamma ||u||_T. \tag{3}$$

Definition III.3 (Passivity): A relation H is said to be *passive* if there exists $\delta \geq 0$ such that, for all $T \geq 0$ and $u \in \mathrm{Do}(H)$

$$\langle u, Hu\rangle_T \geq \delta ||u||_T^2. \tag{4}$$

H is said to be *strictly passive* if (4) holds with $\delta > 0$.

Again, this definition is very similar to the well-known standard definition. It is important to note that, while passivity theory is motivated by the study of energy, the definition and related results continue to hold in the case where no energy interpretation is available.

A. System Relaxation

In studying the passivity of physical systems, it is generally assumed that the system begins in a relaxed state, resulting from applying zero input over an extended period of time.

In the case of an hysteretic system, the output which results from this relaxation process is not unique, since it will depend on the past input history. However, the *set* of such "relaxed states" can be classified in terms of a property of the corresponding

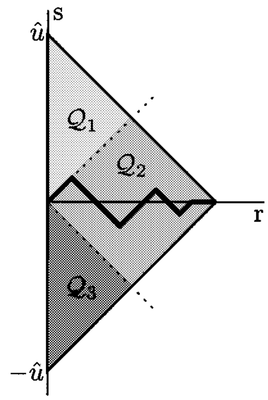

Fig. 6. One example of a zero-input boundary.

Preisach boundaries. Any boundary corresponding to zero input must terminate at the origin of the Preisach plane. Defining the regions

$$\begin{aligned} \mathcal{Q}_1 &= \{(r,s) \in \mathcal{P}_r \,|\, s > r\} \\ \mathcal{Q}_2 &= \{(r,s) \in \mathcal{P}_r \,|\, |s| \le r\} \\ \mathcal{Q}_3 &= \{(r,s) \in \mathcal{P}_r \,|\, s < -r\} \end{aligned}$$

then because of boundary dynamics, zero-input boundaries must be fully contained in $\mathcal{Q}_2$. This is illustrated by example in Fig. 6.

It can be seen that when the system is relaxed (i.e., the input is zero), $\mathcal{Q}_1 \subset \mathcal{P}_-$ and $\mathcal{Q}_3 \subset \mathcal{P}_+$. The state of individual relays in $\mathcal{Q}_2$ depends on the shape of the boundary, and hence on past input history. The output value of the relaxed system is therefore not unique, but lies somewhere on the y-axis inside the major hysteresis loop.

B. Model Passivity

Many hystereses, including classical magnetic hysteresis and the behavior of our SMA actuator, do not satisfy the sector condition for passivity. It is clear that hysteretic systems are, in general, not passive from u to y. In the case of actuators, the input is generally some form of mechanical or electrical force, and the measured output is displacement. Hence, the associated energy pair is not (u, y), but $(u, \dot{y})$. We will show that models for which $\mu \in \mathcal{M}_p$ are passive from u to $\dot{y}$.

First, the energy storage traits of individual relays are determined. This leads to the passivity characteristics of the individual relay weighted pairs $(u, \dot{y}_{r,s})$. Finally, the passivity of $(u, \dot{y})$ in the overall Preisach model will be shown.

Define $q_+ = 2\mu(r,s)(s+r)$ and $q_- = -2\mu(r,s)(s-r)$. If the product $u \cdot \dot{y}_{r,s}$ has units of power, then q_+ and q_- represent the net work required to switch the relay $\gamma_{r,s}$ "on" or "off," respectively. Note that the energy loss in one full cycle is equal to $q_+ + q_- = 4r\mu(r,s)$, the area of the relay. This agrees with the well-known result that the hysteretic energy loss in a magnetic circuit due to periodic excitation is equal to $\oint H\, dB$, the area of the hysteresis loop (e.g., [17]).

Lemma III.1 (Relay Passivity): When a Preisach system for which $\mu \in \mathcal{M}_p$ starts from a relaxed state, $\langle u, \dot{y}_{r,s}\rangle_T \ge 0$ for each relay in $\mathcal{P}_r$ and all $u \in W_1^2[0,T]$.

Proof: Consider an arbitrary input $u \in W_1^2[0,T]$, and define the sets of times at which u causes the relay $\gamma_{r,s}$ to switch

$$\begin{aligned} \mathcal{T}_+ &= \{t \in [0,T] : y_{r,s}(t) \text{ changes from } -\mu(r,s) \text{ to} \mu(r,s)\} \\ \mathcal{T}_- &= \{t \in [0,T] : y_{r,s}(t) \text{ changes from } \mu(r,s) \text{ to } -\mu(r,s)\}. \end{aligned}$$

Let n_+ be the number of points in $\mathcal{T}_+$ and n_- the number in $\mathcal{T}_-$. Assuming first that both n_+ and n_- are finite, it follows that[1]

$$\begin{aligned} \langle u, \dot{y}_{r,s}\rangle_T &= \int_0^T u\dot{y}_{r,s}\, dt \\ &= \int_0^T u \sum_{\tau\in\mathcal{T}_+} 2\mu(r,s)\delta(t-\tau)\, dt \\ &\quad + \int_0^T u \sum_{\tau\in\mathcal{T}_-} -2\mu(r,s)\delta(t-\tau)\, dt \\ &= \sum_{\tau\in\mathcal{T}_+} 2\mu(r,s) \int_0^T u\delta(t-\tau)\, dt \\ &\quad + \sum_{\tau\in\mathcal{T}_-} -2\mu(r,s) \int_0^T u\delta(t-\tau)\, dt \\ &= \sum_{\tau\in\mathcal{T}_+} 2\mu(r,s)u(\tau) + \sum_{\tau\in\mathcal{T}_-} -2\mu(r,s)u(\tau) \\ &= \sum_{\tau\in\mathcal{T}_+} 2\mu(r,s)(s+r) + \sum_{\tau\in\mathcal{T}_-} -2\mu(r,s)(s-r) \\ &= \sum_{\tau\in\mathcal{T}_+} q_+ + \sum_{\tau\in\mathcal{T}_-} q_- \\ &= n_+q_+ + n_-q_-. \end{aligned}$$

The demonstration that $n_+q_+ + n_-q_- \ge 0$ relies on Assumption II.1, and depends on the region in which the relay $\gamma_{r,s}$ resides. In the relaxed state, we have $\mathcal{Q}_1 \subset \mathcal{P}_-$, so if $(r,s) \in \mathcal{Q}_1$ then the first switch must be $-\mu(r,s) \to \mu(r,s)$. Because two consecutive switches cannot be in the same direction, $n_+ \ge n_-$. Under Assumption II.1, $q_+ \ge |q_-|$ in $\mathcal{Q}_1$, so $n_+q_+ + n_-q_- \ge 0$. If $(r,s) \in \mathcal{Q}_2$, the direction of the first switch depends on the past history and evolution of $u(t)$, but $q_+ = 2\mu(r,s)(s+r) \ge 0$ and $q_- = -2\mu(r,s)(s-r) \ge 0$, so $n_+q_+ + n_-q_- \ge 0$. If $(r,s) \in \mathcal{Q}_3$, the argument here is similar to that for $\mathcal{Q}_1$. The first switch must be $\mu(r,s) \to -\mu(r,s)$, so $n_- \ge n_+$. Since $q_- \ge |q_+|$, we have $n_+q_+ + n_-q_- \ge 0$.

If either n_+ or n_- is infinite, then the relay has undergone an infinite number of full cycles, each resulting in a net energy loss equal to the area of the relay. So in this case also, $\langle u, \dot{y}_{r,s}\rangle_T \ge 0$.

Hence, for any $T \ge 0$, arbitrary $u \in W_1^2[0,T]$ and all $(r,s) \in \mathcal{P}_r$, we have $\langle u, \dot{y}_{r,s}\rangle_T \ge 0$ when the system begins in a relaxed state. ■

Theorem III.2 (Model Passivity): Given a Preisach operator Γ, with $\mathrm{Do}(\Gamma) = \{u \in W_1^2[0,T] \,|\, u(0) = 0\}$ and $\mu \in \mathcal{M}_p$, the composite relational operator $(d/dt)\,\Gamma$ is passive.

[1]Note that $y_{r,s}$ is discontinuous at switching times. The integral in $\langle u, \dot{y}_{r,s}\rangle_T$ can still be well defined, however, by interpreting $\dot{y}_{r,s}$ as the delta functional at the switching times.

Proof: The restriction of the domain of Γ ensures that the system begins in a relaxed state, and the passivity of individual relays can be exploited. In this case, we have

$$\begin{aligned}\langle u, \dot{y}\rangle_T &= \int_0^T u\dot{y}\, dt \\ &= \int_0^T u \frac{d}{dt}\left[\int_{\mathcal{P}_r} y_{r,s}(t)\, ds\, dr\right] dt \\ &= \int_0^T u \left[\int_{\mathcal{P}_r} \dot{y}_{r,s}(t)\, ds\, dr\right] dt \\ &= \int_{\mathcal{P}_r} \left[\int_0^T u\dot{y}_{r,s}(t)\, dt\right] ds\, dr. \\ &= \int_{\mathcal{P}_r} \langle u, \dot{y}_{r,s}\rangle_T\, ds\, dr.\end{aligned}$$

The result then follows from Lemma III.1. ■

1) Notes on Passivity: Without the restriction on $\mathrm{Do}(\Gamma)$ to $W_1^2[0,T]$ with $u(0)=0$, one could choose any $T \geq 0$ and construct a continuous input u which was negative and increasing over $[0,T]$. All switches caused by this input would be from $-\mu(r,s)$ to $\mu(r,s)$, and any relay which switched would do so only once. Since $u(t) < 0, q_+ < 0$ for each relay switched, and $\langle u, \dot{y}\rangle_T < 0$. Essentially, this input is extracting energy which is stored in the system, and this underscores the importance of relaxation and minimum energy states when considering system passivity. If the limitation $u(0)=0$ is not enforced, it can be shown that $\langle u, \dot{y}\rangle_T$ is lower bounded by a negative constant [18], a condition sometimes referred to as *weakly passive*. Note that the restriction $u(0)=0$ is simply a mathematical statement of the "relaxation assumption" which is routinely made when discussing input-output passivity. The passivity-based proof of stability in Section V will obviously only hold when the Preisach model input satisfies this specific initial condition. This is not overly restrictive for two reasons:

1) the history of sequential inputs to the system can always be thought of as one input, originating at some point in the past at $u=0$;
2) the control framework investigated in Section V allows for compensation of any input which is initially nonzero.

It is tempting to try to extend passivity to inputs in $L_2[0,T]$. Since $W_1^2[0,T]$ is dense in $L_2[0,T]$, for any $u \in L_2[0,T]$ a sequence $\{u_i\} \subset W_1^2[0,T]$ can be constructed such that $u_i \to u$. One could then use the continuity of the inner product and the result on $W_1^2[0,T]$ to extend passivity to $L_2[0,T]$. However, such an argument requires that the Preisach model itself be continuous on $L_2[0,T]$, when in fact one can construct a counter-example to refute this. Consider for example the sequence of continuous functions

$$u_i(t) = \begin{cases} i - i^2|t-1| & |t-1| \leq \frac{1}{i} \\ 0 & |t-1| > \frac{1}{i}. \end{cases}$$

The impulse $u_i(1) = i$ can erase some of the previous memory, permanently altering the output of the Preisach operator. Although $\lim_{i\to\infty} \|0 - u_i\| = 0$, this amnesia means that $\lim_{i\to\infty} \|\Gamma(0) - \Gamma(u_i)\| \neq 0$. Hence, the Preisach operator is not continuous on $L_2[0,T]$.

IV. Dissipativity of the Preisach Model

In the previous section, passivity was shown using a traditional input–output representation of the Preisach model. In this section, we introduce a state-space representation; by placing the model in a state-space framework, more general stability techniques such as Lyapunov and dissipativity theory may be applied. In particular, we show that the passivity result of the previous section is more easily and elegantly arrived at by applying dissipativity theory in the state-space framework. This result is then used in the next section to construct a stabilizing controller for the differential actuator, based on the Passivity theorem.

A. State-Space Representation

The following definition of a dynamical system is standard (e.g., [19]).

Definition IV.1 (Dynamical System): A dynamical system is defined through the input, output and state spaces $\mathcal{U}, \mathcal{Y}$ and $\mathcal{X}$, as well as the state transition operator $\phi : \mathbb{R}^2 \times \mathcal{X} \times \mathcal{U} \mapsto \mathcal{X}$ and the read-out operator $r : \mathcal{X} \times \mathcal{U} \mapsto \mathcal{Y}$. ϕ must satisfy the standard axioms.

Consistency: $\phi(t_o, t_o, x_o, u) = x_o$ for all $t_o \in \mathbb{R}$, $x_o \in \mathcal{X}, u \in \mathcal{U}$.

Determinism: $\phi(t_1, t_o, x_o, u_1) = \phi(t_1, t_o, x_o, u_2)$ for all $t_o, t_1 \in \mathbb{R}, t_1 \geq t_o, x_o \in \mathcal{X}$, and all $u_1, u_2 \in \mathcal{U}$ satisfying $u_1(t) = u_2(t)$ for all $t_o \leq t \leq t_1$.

Semi-Group: $\phi(t_2, t_o, x_o, u) = \phi(t_2, t_1, \phi(t_1, t_o, x_o, u), u)$ for all $t_o \leq t_1 \leq t_2, x_o \in \mathcal{X}$, $u \in \mathcal{U}$.

Stationarity: $\phi(t_1 + T, t_o + T, x_o, \sigma_T u) = \phi(t_1, t_o, x_o, u)$ for all $t_o \in \mathbb{R}$, $t_1 \geq t_o$, $T \in \mathbb{R}$, $x_o \in \mathcal{X}$, and $u \in \mathcal{U}$. σ_T is the shift operator: $\sigma_T u(t) = u(t+T)$.

For the Preisach model, the input space $\mathcal{U}$ is defined, for some system-dependent $\hat{u} > 0$, as

$$\mathcal{U} = \left\{ u \in C^0(-\infty, \infty) \mid \|u\|_\infty \leq \hat{u} \text{ and } \lim_{t\to-\infty} u(t) = 0 \right\}.$$

The output space $\mathcal{Y}$ is the set of real-valued functions $C^0(-\infty, \infty)$.

For any interval $[t_0, t_1]$ in $\mathbb{R}$, the notation $u_{[t_0,t_1]}$ denotes the restriction of u to $[t_0, t_1]$. The notation $\mathcal{U}[t_0, t_1]$ denotes the set obtained when every element of $\mathcal{U}$ is restricted to $[t_0, t_1]$.

Since the boundary ψ embodies the memory of the model, it is a natural choice for the state. The following definition captures the salient features of the boundaries, and fits Willems' definition of a state space [19].

Definition IV.2 (The State Space): The state space $\mathcal{X}$ is the set of continuous functions $\psi : [0, \hat{u}] \mapsto \mathbb{R}$ which satisfy the following properties:

(BP1) Lipschitz: $|\psi(r_1) - \psi(r_2)| \leq |r_1 - r_2|, \forall r_1, r_2 \in [0, \hat{u}]$;

(BP2) initial condition: $\psi(\hat{u}) = 0$.

The Lipschitz property is more general than required, since boundaries may only be composed of segments of slope ± 1. However, including all functions with Lipschitz constant 1 leads to a complete state space. The proof of this and other functional properties of the state space may be found in [20]. The property (BP2), along with (BP1), ensures that elements of $\mathcal{X}$ are within the triangle defined by $\mathcal{P}_r$.

1) Reduced Memory Sequences: We now introduce the intermediate space $\mathcal{S}$ of *reduced memory sequences*, which will be used in the construction of the state transition operator ϕ.

The wiping out property of the Preisach model was described in Section II. In essence, any input maximum which exceeds previous maxima will remove the memory of those maxima; minima can be similarly "wiped out." At a given time t, only certain past extrema are retained and affect the output. They form an alternating set of input maxima and minima, in which each maximum is smaller in amplitude than the previous one, and each minimum is larger than the previous one. The two series converge to $u(t)$. This alternating sequence is known as the *reduced memory sequence*, and the following mathematical construction is based on that of [15]. A different but equivalent approach to memory sequences in hysteresis operators can be found in [13].

For any $u \in \mathcal{U}(-\infty, T]$ and any $\tau \leq T$, set $s_0 = 0$ and $\eta = \max_{t\in(-\infty,\tau]} |u(t)|$. This is well defined, since $\lim_{t\to-\infty} u(t) = 0$. Let $t_1 = \max\{t \in (-\infty, \tau] \,|\, |u(t)| = \eta\}$, and define the elements $s_i|_{i=1,2,\ldots}$ of the reduced memory sequence $s(u, \tau)$ as follows:

$$\begin{aligned} i = 1{:} \quad & s_1 = u(t_1), \\ s_{i-1} < s_{i-2}{:} \quad & s_i = \max_{t\in(t_{i-1},\tau]} u(t) \\ & t_i = \max\{t \in (t_{i-1}, \tau] \,|\, u(t) = s_i\} \\ s_{i-1} > s_{i-2}{:} \quad & s_i = \min_{t\in(t_{i-1},\tau]} u(t) \\ & t_i = \max\{t \in (t_{i-1}, \tau] \,|\, u(t) = s_i\} \end{aligned} \tag{5}$$

terminating the sequence if $t_i = \tau$.

Note that the values s_i are well defined: by definition of s_{i-1} in (5), $u(t) > s_{i-1}$ (or $u(t) < s_{i-1}$) over $(t_{i-1}, \tau]$. Since u is continuous, the required maximum (or minimum) is well defined. The times t_i are similarly well defined, since the maximum is being taken over a nonempty set and τ is finite. The sequence $\{t_i\}$ is merely used to construct $\{s_i\}$ and then discarded: the time at which extrema occur is of no significance in the Preisach model due to its static nature.

If the input u has a finite number of extrema in $(-\infty, \tau]$, the above sequence has finite length N, $t_N = \tau$ and $u(t_N) = u(\tau)$. In this case, the tail of the sequence is formed by setting $s_i = s_N$ for $i > N$. If the sequence is infinite, then setting $t^* = \sup\{t_i\}$, the input u must be constant over $[t^*, \tau]$. Note that in both cases, $\lim_{i\to\infty} s_i = u(\tau)$.

Define the notation $N(s) = \sup\{i \,|\, s_{i-1} \neq s_i\}$. For any reduced memory sequence $s(u, \tau)$, this is the index beyond which s_i is constant and equal to $u(\tau)$. If u has a finite number of extrema in $(-\infty, \tau]$, then $N(s(u, \tau))$ is finite; otherwise, $N(s)$ may be infinite. Also, for any sequence $s = \{s_i\}$, let s^e and s^o be the even and odd subsequences $s^e = \{s_i\}_{i=2,4,\ldots} = \{s_i^e\}$ and $s^o = \{s_i\}_{i=1,3,\ldots} = \{s_i^o\}$.

Definition IV.3 (Space of Reduced Memory Sequences): The space of reduced memory sequences, $\mathcal{S} \subset l_\infty$, is composed of all sequences s with $\|s\|_\infty \leq \hat{u}$, and for which the even subsequence s^e and odd subsequence s^o satisfy

1) s^e is strictly decreasing (or, respectively, strictly increasing) up to $N(s)$ and, if $N(s) < \infty$, constant thereafter;
2) s^o is strictly increasing (or, respectively, strictly decreasing) up to $N(s)$ and, if $N(s) < \infty$, constant thereafter;
3) $\lim_{i\to\infty} s_i^e = \lim_{i\to\infty} s_i^o$.

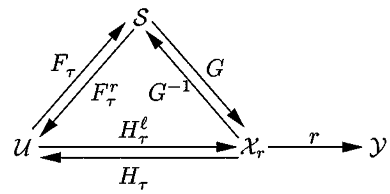

Fig. 7. Relationship between system spaces.

Fig. 7 shows the spaces defined thus far. $\mathcal{X}_r$ is the reachable subset of $\mathcal{X}$: it is shown in [20] that $\mathcal{X}$ is approximately reachable from the initial state ψ^*. That is, the state can be driven arbitrarily close to any element of $\mathcal{X}$ from ψ^* in finite time. We now define mappings between these spaces, and note some of their properties.

$\mathbf{F}_\tau : \mathcal{U} \mapsto \mathcal{S}$

For any time $\tau < \infty$ and any input $u \in \mathcal{U}(-\infty, \tau]$, the reduced memory sequence $F_\tau u = s(u, \tau)$ is defined as in (5).

$\mathbf{F}_\tau^\mathbf{r} : \mathcal{S} \mapsto \mathcal{U}$

The reduced memory sequence $s(u, \tau)$ captures only information regarding dominant extrema of $u_{(-\infty,\tau]}$. There are therefore an infinite number of inputs $u_i \neq u$ which are equivalent, in the sense that they have the same reduced memory sequence: $F_\tau u_i = F_\tau u$. Hence, no inverse of F_τ exists. A right-inverse $F_\tau^r : \mathcal{S} \mapsto \mathcal{U}(-\infty, \tau]$ is defined below.

For any $s(\cdot, \tau) \in \mathcal{S}$, it is required to construct an input $u \in \mathcal{U}(-\infty, \tau]$ with extrema equal to the elements of s and satisfying $u(\tau) = \lim_{i\to\infty} s_i$. Choose any $t_0 < \tau$, and let $\{t_i\}$ be a partition of $[t_0, \tau]$ defined for all $i \geq 1$ by

$$t_i = t_o + \frac{\tau - t_0}{2} \sum_{k=0}^{i-1} \frac{1}{2^k}.$$

Note that $\lim_{i\to\infty} t_i = \tau$. Set $s_0 = 0$ and define $u(t)$ on $(-\infty, \tau]$ by straight-line interpolation between the points (t_i, s_i):

$$u(t) = \begin{cases} 0 & t \leq t_0, \\ s_i & t = t_i, \\ (t - t_{i-1})\frac{s_i - s_{i-1}}{t_i - t_{i-1}} + s_{i-1} & t_{i-1} < t < t_i. \end{cases}$$

The resulting output $u \in \mathcal{U}(-\infty, \tau]$ has extrema corresponding to elements of $s(\cdot, \tau)$, and

$$u(\tau) = u\left(\lim_{i\to\infty} t_i\right) = \lim_{i\to\infty} s_i.$$

Note that $F_\tau F_\tau^r s = s$ for all $s(\cdot, \tau) \in \mathcal{S}$.

$\mathbf{G} : \mathcal{S} \mapsto \mathcal{X}_\mathbf{r}$

Any reduced memory sequence $s(u,\tau) \in \mathcal{S}$ defines a corresponding boundary $\psi = G(s)$. The elements of s correspond to the corners of the boundary curve, as follows: For all $i < \infty$, define the set of points $p_i \in \mathbb{R}^2$

$$
\begin{aligned}
p_0 &= (\hat{u}, 0) \\
p_1 &= (|s_1|, 0) \\
p_i &= \begin{cases} \left(\frac{s_{i-1}-s_i}{2}, \frac{s_{i-1}+s_i}{2}\right), & s_i < s_{i-1} \\ \left(\frac{s_i-s_{i-1}}{2}, \frac{s_i+s_{i-1}}{2}\right), & s_i > s_{i-1} \end{cases}
\end{aligned} \tag{6}
$$

and $G(s)$ to be the linear interpolate between the points p_i. Note that if $N(s) < \infty$ then for all $i > N(s)$, $s_i = s_{i-1} = s_{N(s)}$ and $p_i = (0, s_{N(s)}) = (0, u(\tau))$. If $N(s)$ is infinite, then the boundary $G(s)$ has an infinite number of corners p_i. In this case, since $\lim_{i\to\infty} s_i^e = \lim_{i\to\infty} s_i^o = u(\tau)$, then $\lim_{i\to\infty} p_i = (0, u(\tau))$. In both cases, the result is as expected for the Preisach model: the boundary at time τ intersects the axis $r = 0$ at the point $(0, u(\tau))$: $\psi(\tau, 0) = u(\tau)$.

Note that the range of G, $\mathrm{Ra}(G)$, is the set of all curves $\psi \in \mathcal{X}$ which have a finite or countably infinite number of alternating segments of slope ± 1. This set is not quite $\mathcal{X}$, since $\mathcal{X}$ also contains continuous curves. However, $\mathrm{Ra}(G) = \mathcal{X}_r$ and $\overline{\mathrm{Ra}(G)} = \mathcal{X}$.

$\mathbf{G^{-1}} : \mathcal{X}_\mathbf{r} \mapsto \mathcal{S}$

For every sequence $s \in \mathcal{S}$, the boundary $G(s)$ is unique, by definition of G. Since $\mathrm{Ra}(G) = \mathcal{X}_r$, the inverse mapping $G^{-1} : \mathcal{X}_r \mapsto \mathcal{S}$ exists. The construction of a sequence $s \in \mathcal{S}$ from any boundary in $\psi \in \mathcal{X}_r$ is done by extracting the coordinates of the corners and calculating the corresponding extrema from (6). If the number of corners N is finite, the reduced memory sequence is completed by setting the tail to $s_i = s_N$ for all $i > N$.

$\mathbf{H}_\tau^\ell : \mathcal{U} \mapsto \mathcal{X}_\mathbf{r}$

$\mathbf{H}_\tau : \mathcal{X}_\mathbf{r} \mapsto \mathcal{U}$

The mappings $H_\tau^\ell : \mathcal{U} \mapsto \mathcal{X}_r$ and $H_\tau : \mathcal{X}_r \mapsto \mathcal{U}$ are defined as the compositions $H_\tau = F_\tau^r G^{-1}, H_\tau^\ell = GF_\tau$. H_τ^ℓ is a left-inverse of H_τ, since $H_\tau^\ell H_\tau = GF_\tau F_\tau^r G^{-1} = I$.

2) State Transition and Read-Out Operators: The state transition operator ϕ determines the state $\psi = \phi(t_1, t_o, \psi_o, u)$ which results at time t_1 from applying an input $u \in \mathcal{U}[t_o, t_1]$ to a system starting in state ψ_o at time t_o. For this operation to be well posed, the state ψ_o must be reachable and u must be admissible to ψ_o at t_o; that is, $u(t_o) = \psi(t_o, 0)$ (cf. Definition II.1).

The state transition operator ϕ is defined using the mappings introduced in the previous section. Given some interval $[t_0, t_1]$, some initial state $\psi_o \in \mathcal{X}_r$, and some input $u \in \mathcal{U}[t_0, t_1]$ admissible to ψ_o at t_o, $\phi(t_1, t_o, \psi_o, u)$ is defined as follows:

1) determine the memory sequence for the initial state: $s(\cdot, t_0) = G^{-1}\psi_o$;
2) construct an input $u_o \in \mathcal{U}(-\infty, t_0]$ which generates $s(\cdot, t_0)$: $u_o = F_{t_0}^r s(\cdot, t_0)$;
3) concatenate the inputs u_o and u to form[2] $\tilde{u} = u_o \diamond u \in \mathcal{U}(-\infty, t_1]$;
4) determine the corresponding boundary ψ_1 at time t_1: $\psi_1 = GF_{t_1}\tilde{u}$.

[2] $\diamond$: $C^0 \times C^0 \mapsto C^0$ is the concatenation operator, adding the second argument to the end of the first.

Recalling that $H_\tau = F_\tau^r G^{-1}$ and $H_\tau^\ell = GF_\tau$, the state transition function ϕ is given by

$$\phi(t_1, t_o, \psi_o, u) = H_{t_1}^\ell \left[H_{t_0}\psi_o \diamond u_{(t_0,t_1]}\right].$$

It is shown in [20] that ϕ satisfies the consistency, determinism, semigroup and stationarity axioms.

The read-out function r gives the system output corresponding to a particular state ψ. Recall that the Preisach model output is (1)

$$y(t) = \int_0^\infty \int_{-\infty}^\infty \mu(r,s)\gamma_{r,s}[u](t)\, ds\, dr.$$

Since relays below ψ have output $+1$ and relays above, -1, r can be defined as a function of the state, ψ

$$
\begin{aligned}
y(t) = r(\psi(t)) &= \int_0^\infty \int_{-\infty}^{\psi(t)} \mu(r,s)\, ds\, dr \\
&\quad - \int_0^\infty \int_{\psi(t)}^\infty \mu(r,s)\, ds\, dr.
\end{aligned}
$$

B. Dissipativity

The existence of the state-space formulation of the Preisach model allows the application of dissipativity theory in the analysis and design of closed-loop systems incorporating Preisach hystereses. Dissipativity theory is a generalization of the passivity concepts discussed earlier, and the major input–output stability results can all be cast as special cases of dissipativity theory [19]. Dissipativity is defined in terms of the relationship between two functions known as the *supply rate* and the *storage function*:

Definition IV.4 (Dissipativity [19]): A dynamical system is said to be dissipative with respect to the (locally integrable) *supply rate* $w : U \times Y \mapsto \mathbb{R}$ if there exists a nonnegative function $S : \mathcal{X} \mapsto \mathbb{R}^+$, called the *storage function*, such that for all $t_1 \geq t_o, x_o \in \mathcal{X}$, and $u \in \mathcal{U}$,

$$S(x_o) + \int_{t_o}^{t_1} w(u(t), y(t))\, dt \geq S(\phi(t_1, t_o, x_o, u)). \tag{7}$$

Here, it is shown that the Preisach model is dissipative in a more general sense, since the supply rate will include the derivative of the output. This type of general supply rate has been investigated in [21].

Essentially, for a system to be dissipative, the sum of the storage in the initial state and the supply generated by the input must not be less than the storage in the final state. In other words, there is no internal generation of storage. The word “energy” is conspicuously absent from this description: while dissipativity theory is based on energy concepts, the supply rate and storage function are generalizations of the physical concepts of “rate of energy supply” and “amount of stored energy.” Like passivity, there need not be any physical energy interpretation in order for the definition or related results to hold.

In general, storage functions for physical systems are not unique. However, it is often the case that the actual energy stored in a system is a storage function with some related supply rate. We now derive an expression for the energy stored in the Preisach model, as a function of the state ψ.

In Section III, it was seen that the energy transferred to a relay in a switch from -1 to $+1$ was $q_+ = 2\mu(r,s)(s+r)$; energy transferred from $+1$ to -1 is $q_- = -2\mu(r,s)(s-r)$. Recall the regions $\mathcal{Q}_1, \mathcal{Q}_2$, and $\mathcal{Q}_3$ (cf. Fig. 6). We observe that if $\mu(r,s) \geq 0$, energy transfer is positive for all relay switches except: $q_+ \leq 0$ for relays in $\mathcal{Q}_3$ and $q_- \leq 0$ for relays in $\mathcal{Q}_1$. Negative energy transfer represents energy being recovered from the system: relays whose next switch will result in negative energy transfer are *storing energy*. The formula for total stored energy is

$$Q(\psi(t)) = 2\int_{\mathcal{Q}_1 \cap \mathcal{P}_+(t)} \mu(r,s)(s-r)\,ds\,dr - 2\int_{\mathcal{Q}_3 \cap \mathcal{P}_-(t)} \mu(r,s)(s+r)\,ds\,dr. \tag{8}$$

Recall from their definition in (2) that the regions $\mathcal{P}_+(t)$ and $\mathcal{P}_-(t)$ are entirely defined by the boundary $\psi(t)$. If $\mu \in \mathcal{M}_p$ then $Q(\psi) \geq 0$, since $s > r$ in $\mathcal{Q}_1$ and $s < -r$ in $\mathcal{Q}_3$.

The idea of system relaxation discussed in Section III can now be formalized in terms of this total stored energy.

Proposition IV.1 (Minimum Energy): If $\mu \in \mathcal{M}_p$, then whenever $u(t) = 0$, the Preisach model is in a state of minimum stored energy.

Proof: If $u(t) = 0$, then $\psi(t,0) = 0$. Since boundaries have Lipschitz constant 1, ψ must be entirely contained in the region $\mathcal{Q}_2$. Thus $\mathcal{Q}_1 \cap \mathcal{P}_+ = \emptyset$ and similarly, $\mathcal{Q}_3 \cap \mathcal{P}_- = \emptyset$. So from (8), $Q(\psi) = 0$ since the areas of integration are empty. ■

Regarding this proposition, it is worth making the following points:

- In many physical hystereses, μ is nonzero near the origin of $\mathcal{P}$, making Proposition IV.1 both necessary and sufficient.
- The Preisach boundary of a magnetic material in the minimum-energy demagnetized state corresponds to the initial boundary ψ^* [15], which is consistent with Proposition IV.1.
- The energy diagrams of materials undergoing a phase transition show multiple minima, suggesting many different minimum-energy equilibria (e.g., [13]). Again, this is consistent with Proposition IV.1.

It will now be shown that the Preisach model satisfies the dissipation inequality (7) with the generalized supply rate $w = u\dot{y}$. We do this by demonstrating that the recoverable stored energy Q is a storage function for this supply rate. For a more abstract approach to the topic of dissipation in hysteresis operators, see the work on *hysteresis potentials* in [13].

Theorem IV.2: If $\mu \in \mathcal{M}_p$, the Preisach model satisfies the generalized dissipation inequality

$$Q(\psi_o) + \int_{t_o}^{t_1} u\dot{y}\,dt \geq Q(\psi_1) \tag{9}$$

for any $\psi_o \in \mathcal{X}$, $t_1 \geq t_o$ and $u \in \mathcal{U}[t_o, t_1]$ such that $\phi(t_1, t_o, \psi_o, u) = \psi_1$.

Proof: Recall that if $\mu \in \mathcal{M}_p$ then $Q(\psi) \geq 0$. Also, μ and $\mathcal{P}_r$ are bounded, so $Q(\psi) < \infty$ for all ψ, and $Q : \mathcal{X} \mapsto \mathbb{R}^+$ is a valid storage function. It remains to show that for any initial state ψ_o and $u \in \mathcal{U}[t_o, t_1]$ such that $\psi_1 = \phi(t_1, t_o, \psi_o, u)$, (9) is satisfied.

The remainder of the proof is outlined below; for a detailed proof, see Appendix A. The idea is to consider an arbitrary input u and write out the expression for the energy transferred by u to the system on a relay by relay basis. This gives the expression

$$\int_{t_o}^{t_1} u\dot{y}\,dt = \int_{\mathcal{P}_r} n_+(r,s) q_+\,ds\,dr + \int_{\mathcal{P}_r} n_-(r,s) q_-\,ds\,dr$$

where $n_+(r,s)$ is the number of times the relay $\gamma_{r,s}$ is switched from -1 to $+1$, and $n_-(r,s)$ is the number of times it is switched from $+1$ to -1. We then remove from the right-hand side quantities which are positive: energy transferred when a relay is fully cycled, and positive energy transfer during partial cycling (which depends on the region of $\mathcal{P}_r$). After a nontrivial step in regrouping terms on the right-hand side, this results in

$$\begin{aligned}\int_{t_o}^{t_1} u\dot{y}\,dt \geq{}& 2\int_{\mathcal{Q}_1 \cap \mathcal{P}_+(t_1)} \mu(r,s)(s-r)\,ds\,dr\\ &- 2\int_{\mathcal{Q}_3 \cap \mathcal{P}_-(t_1)} \mu(r,s)(s+r)\,ds\,dr\\ &- 2\int_{\mathcal{Q}_1 \cap \mathcal{P}_+(t_o)} \mu(r,s)(s-r)\,ds\,dr\\ &+ 2\int_{\mathcal{Q}_3 \cap \mathcal{P}_-(t_o)} \mu(r,s)(s+r)\,ds\,dr\\ ={}& Q(\psi(t_1)) - Q(\psi(t_o))\end{aligned}$$

which shows that the dissipation inequality (9) is satisfied. ■

This result provides a distinct advantage over the passivity result of the previous section: one is less constrained in constructing controllers for dissipative systems, when compared to systems which are simply passive. The general idea in design is to match the controller supply rate w_c to that of the plant, w_p, in such a way that the closed-loop system is also dissipative with respect to the storage function $S_c + S_p$. The combined storage function then turns out to be a Lyapunov function for the closed-loop system [19]. Because the form of the supply rate w_c is not constrained, as it is in the Passivity theorem, the designer has a broader family of controller structures to choose from. While this is a worthwhile area to which to devote future efforts, we will be satisfied for now with demonstrating that the result of Section III can be recovered from Theorem IV.2, and then applying the Passivity theorem.

Corollary IV.3 (Preisach Model Passivity): If $\mu \in \mathcal{M}_p$, the composite operator $(d/dt)\Gamma : W_1^2 \mapsto L_2$ is passive.

Recall Definition III.3, and note that the definition of passivity (with $\delta = 0$) is a special case of dissipativity, in which the supply rate is $w = uy$ and the storage function is zero. In order to prove passivity of $(d/dt)\Gamma$, it is required to show that $\int_0^T u\dot{y}\,dt \geq 0$.

Proof: The assumption on μ guarantees that the Preisach model satisfies (9). Suppose the system starts in a state of min-

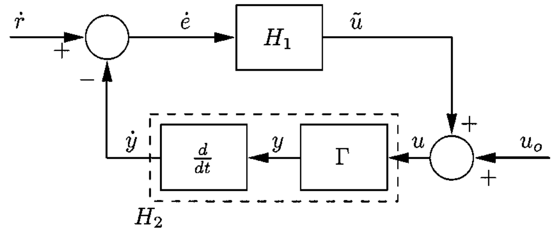

Fig. 9. Control system setup.

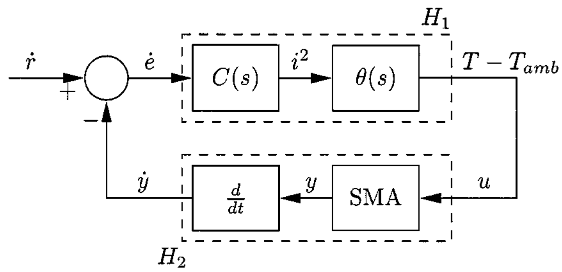

Fig. 10. SMA actuator velocity control configuration.

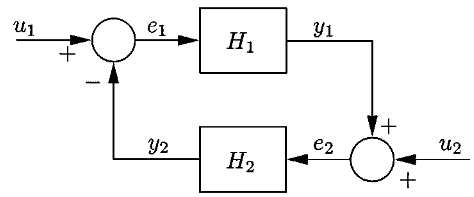

Fig. 8. Standard feedback setup.

imum energy storage, i.e., $u(t_o) = 0$. Then $Q(\psi_o) = 0$ (cf. Proposition IV.1) and from (9),

$$\int_{t_o}^{t_1} u\dot{y}\,dt \geq Q(\phi(t_1, t_o, \psi_o, u)) \geq 0$$

for all $t_1 \geq t_0$ and $u \in \mathrm{Do}(\Gamma)$. In particular, if $t_0 = 0$, we have

$$\int_0^T u\dot{y}\,dt \geq 0$$

which completes the proof. ■

V. Control System Stability

In some applications, a controller is designed which makes use of velocity measurements to achieve its objective. For example, in [22], the authors discuss the damping of vibrations in a flexible beam using piezoceramic actuators. The actuators are bonded to the beam, and the strain which they generate is measured using strain gauges. Measurements of the rate of change of strain are fed back to a proportional-gain controller, which is able to achieve significant damping of beam vibrations.

In this section, we make use the Passivity theorem to define a class of stabilizing controllers for velocity control of Preisach hystereses. This is followed by experimental results of rate control of the rotary SMA actuator described in the introduction.

The following relational definition of feedback stability is similar to that from [23].

Definition V.1 (Stability): The feedback system of Fig. 8, where H_1 and H_2 are relations, will be called stable if, for each $u_1 \in \mathrm{Do}(H_1)$ and $u_2 \in \mathrm{Do}(H_2)$ there exist finite k_e, k_y independent of T such that for $i = 1, 2$ and $\forall T \geq 0$, $\|e_i\|_T \leq k_e(\|u_1\|_T + \|u_2\|_T)$ and $\|y_i\|_T \leq k_y(\|u_1\|_T + \|u_2\|_T)$.

The control setup under consideration is illustrated in Fig. 9. The input $\dot{r}$ is a reference velocity to be tracked, $\dot{e}$ the velocity tracking error, $\tilde{u}$ the control signal, u the Preisach input, y the actuator position output, and $\dot{y}$ the actuator velocity. In order to ensure the consistency of internal signals, we assume that the system H_1 is a relation on $L_2[0,T]$, that it has some smoothing properties so that $\mathrm{Ra}(H_1) \subset W_1^2[0,T]$, and also $\dot{r} \in L_2[0,T]$, $u_o \in \mathrm{Do}(H_2)$.

The input u_o can be used to offset any initial conditions on $\tilde{u}$ and any change in ambient conditions which may have occurred between identification and the current operation conditions. This ensures that the input to the Preisach model is initially zero, so that $u \in \mathrm{Do}(H_2)$ and the passivity of H_2 can be exploited.

Theorem V.1 (Stability): If $H_1 : \dot{e} \mapsto \tilde{u}$ is strictly passive with finite gain and $u_o = -u(0)$, the feedback system of Fig. 9 is stable for all $\dot{r} \in L_2[0,T]$ and any Preisach hysteresis Γ with $\mu \in \mathcal{M}_p$.

Proof: From the system setup, and the hypothesis on u_o, we have $u(0) = 0$ so $u \in \mathrm{Do}(H_2)$. Then, by Theorem III.2, H_2 is passive. The remainder of the proof is along the lines of the standard proof of the passivity theorem (e.g., [23]), although care must be taken to ensure correctness in a relational context (cf. [24]). ■

VI. Experimental Results

In [5], a Preisach model was identified for the SMA actuator described in the introduction. Here, experimental velocity control results for this actuator are presented. The control configuration is shown in Fig. 10. $\theta(s)$ is a common (e.g., [25]) first-order linear approximation for the relationship between electrical power input (i^2) and wire temperature above ambient. Details on the parameters involved in this model and their particular values for the wires used can be found in [25]. The transfer function is $\theta(s) = 61.115/(0.330 + s)$. $C(s)$ is a PD controller $K_p + K_d s$, with $K_p = 1$, $K_d = 0.001$, giving an overall transfer function for $H_1(s)$:

$$H_1(s) = 0.061\frac{s + 1000}{s + 0.330}.$$

This transfer function is strictly passive with finite gain $(\gamma = 185, \delta = 0.061)$. Initially, the wire is at ambient temperature, so no offsetting input u_o is required to satisfy the conditions of Theorem V.1.

The velocity control results are shown in Fig. 11. The sinusoidal reference velocity is shown on the left (solid), along with the measured actuator velocity (dotted). It can be seen that tracking is reasonably good, and that the velocity is stable as expected.

In a memoryless system, a sinusoidal velocity results in a cosinusoidal position signal. In the case of the rotary actuator, there is evident drift in the position response. With SMA actuators subject to periodic inputs, it is common to observe some initial variation before the actuator position settles into a periodic response. This is commonly attributed to initial variations

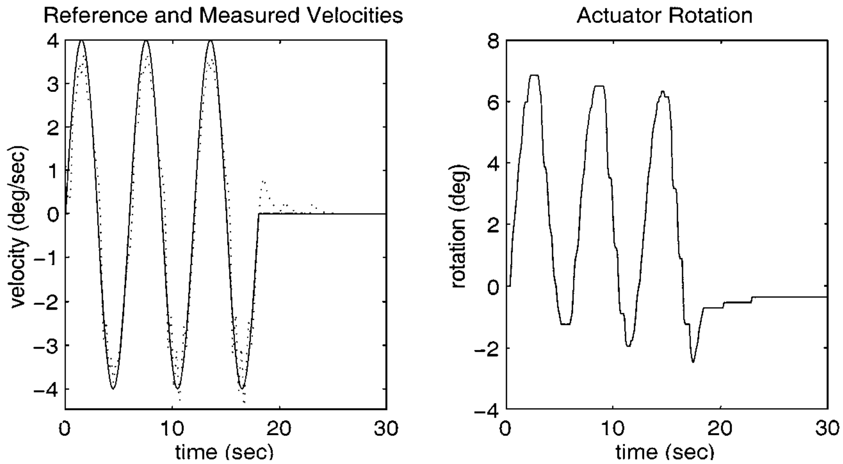

Fig. 11. SMA actuator velocity control response.

in wire heating, an explanation which is generally acceptable for high-frequency excitation. Given the low frequency in this experiment, however, this is an unlikely explanation: the wire should have time to heat fully in the first cycle. More probably, the drift is a result of some residual memory present in the device when the control signal is first applied: although $u(0) = 0$, the output is not initially "zero." The initial output is measured as zero in Fig. 11 due to the incremental nature of the encoder used. This phenomenon can be avoided by subjecting the actuator to an "initializing" input to remove any residual memory, as was done in [8].

VII. Conclusion and Future Work

This work investigated the energy properties of the Preisach hysteresis model; specifically, as these properties relate to control of Preisach hystereses. Using a state-space Preisach model representation, a formula for the energy stored in any given state was derived. This formula suggested a definition for minimum energy states which corresponds with our intuitive concepts. Using these energy results, passivity was demonstrated for the relationship between the input and the derivative of the output for the Preisach model of hysteresis. This leads to stability of rate control of hysteretic systems, if the controller is strictly passive with finite gain. Experimental results were provided which demonstrate the stability of rate feedback control for an SMA actuator. Ongoing research concentrates on extending the present stability results to position control using dissipativity as opposed to passivity, and investigating the effect of time-varying stress on Preisach model validity.

It should be noted also that the Preisach model represents only static hystereses. While this is sufficient for smart actuators such as piezoceramics or SMA in their useful bandwidth, other actuator systems may have some associated dynamics. In some cases, it may still be possible to apply the present results by incorporating actuator dynamics elsewhere in the control loop, as was done here for the heating dynamics associated with the SMA actuator. In general, however, this may not be possible, and the study of such coupled systems is another area of potential future research.

Appendix I
Proof of Theorem IV.2

The full proof of dissipativity requires the following definition of the characteristic function of a set.

Definition A.1—Characteristic Function (e.g., [26]): If A is a subset of a space X, the characteristic function of A, $\chi_A : X \mapsto \{0, 1\}$ is defined by

$$\chi_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A. \end{cases}$$

The proof of the following Lemma is straightforward.

Lemma A.1: Let A and B be subsets of a space X. Then for all $x \in X$

1) if $A \cap B = \emptyset$, then $\chi_A(x) + \chi_B(x) = \chi_{A\cup B}(x)$;
2) if $B \subset A$, then $\chi_A(x) - \chi_B(x) = \chi_{A\sim B}(x)$;

where $\sim$ indicates the set difference operation.

Theorem IV.2: If $\mu \in \mathcal{M}_p$, the Preisach model satisfies the generalized dissipation inequality

$$Q(\psi_o) + \int_{t_o}^{t_1} u\dot{y}\, dt \geq Q(\psi_1) \tag{10}$$

for any $\psi_o \in \mathcal{X}$, $t_1 \geq t_o$ and $u \in \mathcal{U}[t_o, t_1]$ such that $\phi(t_1, t_o, \psi_o, u) = \psi_1$.

Proof: The recoverable stored energy, for any $\psi \in \mathcal{X}$, is

$$Q(\psi(t)) = 2\int_{\mathcal{Q}_1 \cap \mathcal{P}_+(t)} \mu(r,s)(s-r)\, ds\, dr - 2\int_{\mathcal{Q}_3 \cap \mathcal{P}_-(t)} \mu(r,s)(s+r)\, ds\, dr.$$

If $\mu \in \mathcal{M}_p$ then $Q(\psi) \geq 0$, since $s > r$ in $\mathcal{Q}_1$ and $s < -r$ in $\mathcal{Q}_3$. Also, μ and $\mathcal{P}_r$ are bounded, so $Q(\psi) < \infty$ for all ψ, and $Q : \mathcal{X} \mapsto \mathbb{R}^+$ is a valid storage function. It remains to show that for any initial state ψ_o and $u \in \mathcal{U}[t_o, t_1]$ such that $\psi_1 = \phi(t_1, t_o, \psi_o, u)$, (10) is satisfied.

For each relay $\gamma_{r,s}$, let $n_{r,s}$ be the number of times that relay is fully cycled (switched twice) by u. The energy transferred in a switch from -1 to $+1$ is $q_+ = 2\mu(r,s)(s+r)$; in a switch from $+1$ to -1, $q_- = 2\mu(r,s)(r-s)$. The energy transferred in one full cycle is equal to the area of the relay, $4r\mu(r,s) \geq 0$, and represents a net energy loss. Suppose first that all $n_{r,s}$ are finite. The case where $n_{r,s}$ is infinite for some $\gamma_{r,s}$ is discussed at the end of the proof.

Let $\Omega_\pm$ be the collection of relays which are switched an even number of times $(2n_{r,s})$ by u. The energy transfer for each relay in this region is $4rn_{r,s}\mu(r,s) \geq 0$. Denote by $Q_\pm \geq 0$ the *total* energy transferred to *all* relays in $\Omega_\pm$.

Let Ω_+ be the collection of relays which are switched an odd number of times $(2n_{r,s}+1)$ by u, and whose last switch was from -1 to $+1$. The energy transferred to each of these relays is $4rn_{r,s}\mu(r,s) + q_+$. Let $Q_+ \geq 0$ be the total energy transferred to relays in Ω_+ after each has been fully cycled $n_{r,s}$ times. Each relay then undergoes one final switch from -1 to $+1$, so that the total energy transfer to relays in Ω_+ is

$$Q_+ + \int_{\Omega_+} q_+ \, ds \, dr.$$

Similarly, define Ω_- as the collection of relays which are switched an odd number of times $(2n_{r,s}+1)$ by u, and whose last switch was from $+1$ to -1. Let $Q_- \geq 0$ be the total energy transferred to relays in Ω_- after each has been fully cycled $n_{r,s}$ times. The energy transfer for the final switch of each relay in Ω_- is q_-, so the total energy transferred by u to relays in Ω_- is

$$Q_- + \int_{\Omega_-} q_- \, ds \, dr.$$

The total energy transfer from t_o to t_1 is

$$\begin{aligned}\int_{t_o}^{t_1} u\dot{y}\, dt &= Q_\pm + Q_+ + \int_{\Omega_+} q_+ \, ds\, dr + Q_- \\ &\quad + \int_{\Omega_-} q_- \, ds \, dr \\ &\geq \int_{\Omega_+} q_+ \, ds\, dr + \int_{\Omega_-} q_- \, ds\, dr.\end{aligned}$$

In $\mathcal{Q}_2$, both q_+ and q_- are nonnegative, so

$$\begin{aligned}\int_{t_o}^{t_1} u\dot{y}\, dt &\geq \int_{\Omega_+\cap\mathcal{Q}_1} q_+ \, ds\, dr + \int_{\Omega_+\cap\mathcal{Q}_3} q_+ \, ds\, dr \\ &\quad + \int_{\Omega_-\cap\mathcal{Q}_1} q_- \, ds\, dr + \int_{\Omega_-\cap\mathcal{Q}_3} q_- \, ds\, dr \\ &= \int_{\Omega_+\cap\mathcal{Q}_1} 2\mu(r,s)(s+r)\, ds\, dr \\ &\quad + \int_{\Omega_+\cap\mathcal{Q}_3} 2\mu(r,s)(s+r)\, ds\, dr \\ &\quad - \int_{\Omega_-\cap\mathcal{Q}_1} 2\mu(r,s)(s-r)\, ds\, dr \\ &\quad - \int_{\Omega_-\cap\mathcal{Q}_3} 2\mu(r,s)(s-r)\, ds\, dr.\end{aligned}$$

Since $r \geq 0$, $(s+r) \geq (s-r)$ and the first and last terms above can be replaced in the inequality:

$$\begin{aligned}\int_{t_o}^{t_1} u\dot{y}\, dt \geq &\int_{\Omega_+\cap\mathcal{Q}_1} 2\mu(r,s)(s-r)\, ds\, dr \\ &+ \int_{\Omega_+\cap\mathcal{Q}_3} 2\mu(r,s)(s+r)\, ds\, dr \\ &- \int_{\Omega_-\cap\mathcal{Q}_1} 2\mu(r,s)(s-r)\, ds\, dr \\ &- \int_{\Omega_-\cap\mathcal{Q}_3} 2\mu(r,s)(s+r)\, ds\, dr.\end{aligned}$$

Introducing the characteristic functions for Ω_+ and Ω_-, this is then written

$$\begin{aligned}\int_{t_o}^{t_1} u\dot{y}\, dt &\geq 2\mu(r,s)\left[\int_{\mathcal{Q}_1}(s-r)\chi_{\Omega_+}\, ds\, dr\right. \\ &\quad + \int_{\mathcal{Q}_3}(s+r)\chi_{\Omega_+}\, ds\, dr \\ &\quad - \int_{\mathcal{Q}_1}(s-r)\chi_{\Omega_-}\, ds\, dr \\ &\quad \left. - \int_{\mathcal{Q}_3}(s+r)\chi_{\Omega_-}\, ds\, dr\right] \\ &= 2\mu(r,s)\left[\int_{\mathcal{Q}_1}(s-r)\left(\chi_{\Omega_+}-\chi_{\Omega_-}\right)\, ds\, dr\right. \\ &\quad \left. - \int_{\mathcal{Q}_3}(s+r)\left(\chi_{\Omega_-}-\chi_{\Omega_+}\right)\, ds\, dr\right]. \qquad (11)\end{aligned}$$

But Ω_+ contains all the relays $\gamma_{r,s}$ which were in $\mathcal{P}_-$ at t_o and $\mathcal{P}_+$ at t_1. Similarly, Ω_- is exactly all those relays which were switched from $\mathcal{P}_+(t_o)$ to $\mathcal{P}_-(t_1)$. Then $\mathcal{P}_+(t_1)$, the collection of relays which are in the $+1$ state at t_1, can be written as "all relays which started in the $+1$ state, plus those which were switched from -1 to $+1$, less those which were switched to -1:" $\mathcal{P}_+(t_1) = [\mathcal{P}_+(t_o) \cup \Omega_+] \sim \Omega_-$. But $\Omega_- \subset \mathcal{P}_+(t_o)$ (and hence $\mathcal{P}_+(t_o)\cup\Omega_+$), and $\mathcal{P}_+(t_o)\cap\Omega_+ = \emptyset$, so by Lemma A.1

$$\begin{aligned}\chi_{\mathcal{P}_+(t_1)} &= \chi_{[\mathcal{P}_+(t_o)\cup\Omega_+]\sim\Omega_-} \\ &= \chi_{\mathcal{P}_+(t_o)\cup\Omega_+} - \chi_{\Omega_-} \\ &= \chi_{\mathcal{P}_+(t_o)} + \chi_{\Omega_+} - \chi_{\Omega_-}\end{aligned}$$

and $\chi_{\Omega_+} - \chi_{\Omega_-} = \chi_{\mathcal{P}_+(t_1)} - \chi_{\mathcal{P}_+(t_o)}$. Similarly, $\mathcal{P}_-(t_1) = [\mathcal{P}_-(t_o)\cup\Omega_-] \sim \Omega_+$, and $\chi_{\Omega_-} - \chi_{\Omega_+} = \chi_{\mathcal{P}_-(t_1)} - \chi_{\mathcal{P}_-(t_o)}$. Substituting in (11) gives

$$\begin{aligned}&\int_{t_o}^{t_1} u\dot{y}\, dt \\ &\geq 2\mu(r,s)\left[\int_{\mathcal{Q}_1}(s-r)\left(\chi_{\mathcal{P}_+(t_1)}-\chi_{\mathcal{P}_+(t_o)}\right)\, ds\, dr\right. \\ &\quad \left. - \int_{\mathcal{Q}_3}(s+r)\left(\chi_{\mathcal{P}_-(t_1)}-\chi_{\mathcal{P}_-(t_o)}\right)\, ds\, dr\right] \\ &= 2\mu(r,s)\left[\int_{\mathcal{Q}_1}(s-r)\chi_{\mathcal{P}_+(t_1)}\, ds\, dr\right. \\ &\quad - \int_{\mathcal{Q}_3}(s+r)\chi_{\mathcal{P}_-(t_1)}\, ds\, dr \\ &\quad - \left(\int_{\mathcal{Q}_1}(s-r)\chi_{\mathcal{P}_+(t_o)}\, ds\, dr\right. \\ &\quad \left.\left. - \int_{\mathcal{Q}_3}(s+r)\chi_{\mathcal{P}_-(t_o)}\, ds\, dr\right)\right] \\ &= 2\mu(r,s)\left[\int_{\mathcal{Q}_1\cap\mathcal{P}_+(t_1)}(s-r)\, ds\, dr\right. \\ &\quad - \int_{\mathcal{Q}_3\cap\mathcal{P}_-(t_1)}(s+r)\, ds\, dr \\ &\quad - \left(\int_{\mathcal{Q}_1\cap\mathcal{P}_+(t_o)}(s-r)\, ds\, dr\right. \\ &\quad \left.\left. - \int_{\mathcal{Q}_3\cap\mathcal{P}_-(t_o)}(s+r)\, ds\, dr\right)\right] \\ &= Q(\psi_1) - Q(\psi_o).\end{aligned}$$

So the dissipation inequality (9) is satisfied.

If $n_{r,s}$ is infinite for any $\gamma_{r,s}$, then that relay undergoes an infinite number of full cycles. Since energy is lost in each full cycle, the total energy transfer is positive and infinite. But $Q(\psi)$ is bounded for every ψ, so the dissipativity inequality (10) still holds. The Preisach model is dissipative with respect to the supply rate $w(u,y) = u\dot{y}$. ■

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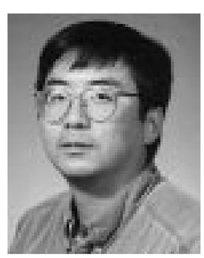

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