MATH 249, WINTER 2017, ASSIGNMENT 8

This assignment is due Friday March 31 at 3pm. It should be submitted using crowdmark. The crowdmark instructions are the same as before.

Questions.

(1) (a) **(4 points)** Suppose I have a set $J \subseteq \mathbb{Z}_{\geq 0}$ and for each $j \in J$ I also have sets $I_1^{(j)}, I_2^{(j)}, \ldots, I_j^{(j)} \subseteq \mathbb{Z}_{\geq 1}$. What is the generating series for compositions where the number of parts must be an element of J and when there are j parts, the ith part must be an element of $I_i^{(j)}$? *Now you can answer all math 239 composition questions. Note that you won't be able to get a closed form in general, though particular math 239-type questions will typically be special cases for which you can.*

(b) **(3 points BONUS)** Describe a class of compositions which are not a special case of the previous part but for which you can still find the generating series.

(2) **(6 points)**

(a) Pick a combinatorial class counted by the Catalan numbers and define it. *Try Wikipedia for some good ideas, but once you pick one, try to do the rest of this problem yourself.*

(b) Describe a decomposition of your class of the form

$$\mathcal{C} \cong \mathcal{X} \cup (\mathcal{Z} \times \mathcal{C}^2)$$

where $\mathcal{X}$ and $\mathcal{Z}$ are both single element classes.

(c) Describe a decomposition of your class of the form

$$\mathcal{C} \cong \sum_{i \geq 0} (\mathcal{Z} \times \mathcal{C})^i$$

where $\mathcal{Z}$ is a single element classes.

(d) Check that you get the same generating function in either case.

(3) **(5 points)**

(a) Use generating series to find the number b_n of binary strings of length n in which every block of 0s is followed by a block of 1s of the same length. *Go through all the steps, describe the decompositition, find the corresponding equation for the generating series, find a closed form for the generating series, calculate $[x^n]B(x)$, using partial fractions if necessary, to get a closed form for the answer.*

(b) Use a combinatorial argument to prove a recurrence for b_n. Use the recurrence to get the closed form for the generating series in a different way.

(4) **(5 points)** Find the generating function for lattice walks using the steps $\uparrow$, $\rightarrow$, $\downarrow$, starting at $(0, 0)$ and which never visit the same vertex twice in the same walk (so in the graph theory sense these will be paths).

(5) **(5 points)** Solve the recurrence $b_n = -3b_{n-1} + 4b_{n-3}$ for $n \geq 3$ with initial conditions $b_0 = 9$, $b_1 = -9$, $b_2 = 18$.