MATH 249, WINTER 2017, ASSIGNMENT 7

This assignment is due Friday March 24 at 3pm. It should be submitted using crowdmark. The crowdmark instructions are the same as before.

Questions.

(1) **(2 points)** This is a little algebraic fact that will be useful to you (on this very assignment even)

Prove that for integer $n > 0$

$$\binom{1/2}{n}(-4)^n = \frac{-2}{n}\binom{2(n-1)}{n-1}$$

(2) **(4 points)** Prove that for formal power series $A(x)$ and $B(x)$, $(AB)'(x) = A'(x)B(x) + A(x)B'(x)$.

(3) **(4 points)** Let a_n be the number of binary strings of length n where every even block of 0s is followed by exactly one 1 and every odd block of 0s is followed by exactly two 1s. Show that

$$\sum_{n\geq 0} a_n x^n = \frac{1+x}{1-x^2-2x^3}$$

(4) **(7 points)** Consider the class of ordered rooted trees where each vertex has either 0 or two children.

(a) Find a functional equation satisfied by the generating series for this class of trees.

(b) Solve the functional equation to obtain a closed form for the generating series of this class.

(c) Use the generalized binomial theorem to obtain a formula for the number of trees in this class of size n. Simplify it so that there are no binomial coefficients with negative or fractional arguments.

(5) (a) **(3 points)** Let a_n be the number of compositions of size n in which every part is at most 3. Give a closed form for the generating series $A(x) = \sum_{n=0}^{\infty} a_n x^n$.

(b) **(3 points)** Let b_n be the number of compositions of size n in which every part is a positive integer that is not divisible by 3. Give a closed form for the generating series $B(x) = \sum_{n=0}^{\infty} b_n x^n$.

(c) **(2 points)** Use the generating series from the previous parts to prove that $b_n = a_n - a_{n-3}$.

(d) **(3 points BONUS)** Give a combinatorial argument for the result of the previous part.