MATH 249, WINTER 2017, ASSIGNMENT 2

This assignment is due Friday January 20 at 3pm. It should be submitted using crowdmark.

Crowdmark instructions.

- You will have recieved an email from crowdmark giving you a link where you can submit – don't use someone else's link as that will replace their assignment with yours and submit nothing for you. Send me an email if you need a new link.
- You will receive 0 if the image quality for your assignment is not adequate. Check after uploading to see if it is clear. If in doubt use a computer lab scanner. It is possible to get an adequate quality image from most cell phone cameras but it is not always easy – this is one of the biggest causes of problems.
- If you have any problems email me **before** the assignment is due. If you do not contact me before the deadline then you will get 0 for a late assignment. If you do contact me in advance your need for a late submission will be dealt with on a case-by-case basis.

Questions.

(1) **(6 points)** In class we defined a bridge as an edge cut of size 1. In the MATH 239 notes (which you can download from LEARN) they define a bridge as an edge e of a graph G for which $G\backslash e$ has more components than G. Prove that these two definitions are equivalent. *Note that $G\backslash e$ means that graph with vertex set $V(G)$ and edge set $E(G)\backslash\{e\}$.*

(2) **(5 points)** Let G be a graph with no bridges. Consider the following relation $\sim$ defined on the set $E(G)$: for $e, f \in E(G)$, $e \sim f$ means that every cycle in G that contains e also contains f.
   (a) Show that $\sim$ is transitive.
   (b) Show that $\sim$ is symmetric.
   (c) Show that $\sim$ is an equivalence relation. The equivalence classes for this relation are called the series classes of $E(G)$.

(3) **(4 points)** Prove that every tree is bipartite.

(4) **(5 points)** Let G be a graph in which every vertex has degree at least $d \geq 2$. Show that G contains a cycle with at least $d + 1$ edges.

(5) (a) **(5 points)** Prove that every automorphism of a tree either fixes a vertex or swaps a pair of adjacent vertices.
   (b) **(3 points BONUS)** What is the largest number of automorphisms (as a function of n) that a tree with n vertices can have?