MATH 249, WINTER 2017, ASSIGNMENT 1

This assignment is due Friday January 13 at 3pm. It should be submitted using crowdmark.

Crowdmark instructions.

- You will have recieved an email from crowdmark giving you a link where you can submit – don't use someone else's link as that will replace their assignment with yours and submit nothing for you.
- You will receive 0 if the image quality for your assignment is not adequate. Check after uploading to see if it is clear. If in doubt use a computer lab scanner. It is possible to get an adequate quality image from most cell phone cameras but it is not always easy – this is one of the biggest causes of problems.
- If you have any problems email me **before** the assignment is due. If you do not contact me before the deadline then you will get 0 for a late assignment. If you do contact me in advance your need for a late submission will be dealt with on a case-by-case basis.

Questions.

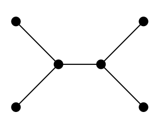
(1) **(4 points)** Find all 4-regular graphs with 8 vertices and justify that your list is complete and all graphs in your list are pairwise non-isomorphic.

(2) **(5 points)** (This is section 4.4 question 5 from the MATH 239 notes) The line graph $L(G)$ of a graph G is the graph whose vertex set is $E(G)$ and in which two vertices are adjacent if and only if the corresponding edges of G are incident with a common vertex.

  (a) Find a graph G such that $L(G)$ is isomorphic to G

  (b) Find non-isomorphic graphs G and G' such that $L(G)$ is isomorphic to $L(G')$.

  (c) If G is



  find $L(G)$, $L(L(G))$, and $L(L(L(G)))$.

(3) **(5 points)** Prove that a circulant graph on n vertices ($n > 2$), has at least $2n$ distinct automorphisms.

(4) **(6 points)** Give examples of graphs with the following properties or explain why they are impossible.

  (a) A 3-regular graph with at least one bridge.

  (b) A bipartite regular graph with 15 vertices and at least one edge.

  (c) A 4-regular graph with 16 vertices and no cycles of length 3 or 4.

(5) **(3 points)** Let $f : G \to H$ be a graph homomorphism. For $v \in V(H)$, the *fibre* of f at v, written $f^{-1}(v)$, is the set $\{w \in V(G) : f(w) = v\}$.

What can you say about the graph induced by $f^{-1}(v)$ in G?

(6) A toroidal grid graph is a graph defined as follows. Begin with k, ℓ, m integers with $k, m \geq 3$ and $\ell \geq 0$. Consider the integer lattice points in the first quadrant of the cartesian plane and identify two points if they differ by $(k, 0)$ or (ℓ, m). These equivalence classes of points are the vertices of the toroidal grid graph and two such vertices are joined by an edge if they have representative points at distance 1 from each other.

(a) **(2 points)** Draw the toroidal grid defined by $k = 3, \ell = 0, m = 3$ and the toroidal grid defined by $k = 3, \ell = 1, m = 4$.

(b) **(3 points BONUS)** Which toroidal grids are circulants?