COMBINATORICS OF FEYNMAN DIAGRAMS LECTURE 35 SUMMARY

WINTER 2018

Summary

Today we talked about weight and weight drop.

From the shape of the initial integrations discussed last time we see that twice the weight of the numerator doesn't increase. Also one of the edges gets set to 1, so for a ϕ^4 graph with 4 external edges and loop number ℓ there remain $2\ell - 3$ integrations and so that is the maximal weight. This maximal weight is achieved for many graphs including the zigzag family. A graph whose period is not maximal weight we'll say has *weight drop*. In Brown's algorithm weight drop comes from when the denominator is a square at some stage. We can use this as an alternate definition of weight drop when restricted to graphs for which the algorithm works. For general graphs you need to be more sophisticated to define weight anyway.

Now if our graph has a 2-separation with 2 external edges on each side then the period is the product of the periods of the two minors obtained from each side with an edge joining the separation vertices. You can prove this by a change of variables. As a corollary, let G be the big graph, G_1 and G_2 the minors for the two sides with the extra edge and let ℓ, ℓ_1, and ℓ_2 be the loop numbers respectively. Then using wt for the weight we have

$$\mathrm{wt}(G_1) \leq 2\ell_1 - 3 \quad \text{and} \quad \mathrm{wt}(G_2) \leq 2\ell_2 - 3,$$

but $\ell = \ell_1 + \ell_2 - 1$ because you remove one from the dimension of the cycle space for each of G_1 and G_2 when you remove the extra edges joining the separation vertices and then you add one to the dimension when you join them into G. So

$$\mathrm{wt}(G) = \mathrm{wt}(G_1) + \mathrm{wt}(G_2) \leq 2(\ell_1 + \ell_2) - 6 = 2\ell + 2 - 6 = 2\ell - 4 < 2\ell - 3,$$

so G has weight drop.

Another tool for weight drop is double triangle reduction. Suppose you have two graphs. One has two triangles joined by an edge. If you contract that edge, add a new edge joining the other ends of the two double edges you created and then remove the duplicates in the two double edges, you get a graph with one triangle where the original graph had two. Now, we calculated that the denominators of these two graphs agree after 7 and 5 integrations respectively. This means that the one will have weight drop iff the other does. The calculation is all about spanning forest polynomials and is combinatorial.

Next time

Next time some of you will give presentations as part of your projects and I'll bring something to eat.

Thanks for coming everyone! Please come to the presentations.

References

See arXiv:0910.5429. The double triangle proof is on page 21-23.