COMBINATORICS OF FEYNMAN DIAGRAMS LECTURE 29 SUMMARY – PART 1

WINTER 2018

Summary

Today we finished our discussion of DSEs with a very brief discussion of my chord diagram expansion results.

The first thing we did was a calculation to convert

$$G(x, L) = 1 - \frac{x}{q^2}\int d^4x \frac{k \cdot q}{k^2 G(x, \log(k^2/\mu^2))(k+q)^2} - \text{same integrand}\Big|_{q^2=\mu^2}$$

where $L = \log(q^2/\mu^2)$ into a pseudo-differential form:

$$G(x, L) = 1 - xG(x, d/d(-\rho))^{-1}(e^{-L\rho} - 1)F(\rho)|_{\rho=0}$$

where $F(\rho)$ is the Feynman integral of the primitive regularized by raising the propagator we insert into to the power $1 + \rho$. In this particular case it is

$$F(\rho) = \int d^4x \frac{k \cdot q}{(k^2)^{1+\rho}(k+q)^2}\Big|_{q^2=1}.$$

You can find the calculation on p72 of the reference linked after lecture 19. Notice how the regularization happens automatically rather than being added in by hand.

From a more combinatorial perspective notice how if we think of $F(\rho)$ as a formal Laurent series (with a simple pole) and $G(x, L)$ as a bivariate formal power series then this equation recursively determines the coefficients of $G(x, L)$ in terms of the coefficients of $F(\rho)$. However it is not determined in a very nice or easy to work with way.

Here's a result that gives a combinatorial solution:

$$G(x, L) = 1 - \sum_C \left(\sum_{i=1}^{t_1} \frac{(-L)^i}{i!} f_{t_1-i}\right) x^{|C|} f_0^{|C|-k} f_{t_k - t_{k-1}} \cdots f_{t_2-t_1}$$

where $F(\rho) = f_0\rho^{-1} + f_1 + f_2\rho + \cdots$ and the sum is over rooted connected chord diagrams C. It remains to explain the t_i; $t_1 < t_2 < \cdots < t_k$ are the indices of the terminal chords of C in recursive order. This is not a standard chord diagram definition but rather something that came up in order to get this result. The recursive chord order is defined as follows. The root is first. Remove the root and let the connected components be $C_1, C_2, \ldots$ ordered by (and rooted at) their first end point running counterclockwise from the original root. Next in the order come the chords of C_1 also ordered by this recursive process, then all the chords of C_2 etc. The terminal chords are the chords which do not cross any chord which comes after them in this order.

Next time

The class continued by starting the next section of the course.

References

The chord diagram expansion discussed here is joint with my then PhD student Nicolas Marie, see arXiv:1210.5457. I have a number of different follow ups with different coauthors (Markus Hihn, Julien Courtiel, and Noam Zeilberger) and some of my current students are also working on related things.