COMBINATORICS OF FEYNMAN DIAGRAMS LECTURE 28 SUMMARY

WINTER 2018

Summary

Today we looked at physical DSEs.

Remember $Z[J] = \int D\phi e^{i\int d^4x \mathcal{L} + J\phi}$. Now if this is supposed to be behaving like an integral then we should have

$$\begin{aligned} 0 &= \int D\phi \frac{\delta}{\delta\phi} e^{i\int d^4x\mathcal{L}+J\phi} \\ &= \int D\phi i \left(\frac{\delta S}{\delta\phi} + J\right) e^{i\int d^4x\mathcal{L}+J\phi} \end{aligned}$$

by the chain rule. Now we can pull that J out of the integral and we'd like to do the same for the other term. To do so use a trick: replace the ϕs by $\delta/\delta J$ which will bring down a ϕ from the exponential. So we have

$$0 = i\left(\frac{\delta S}{\delta\phi}\bigg|_{\phi=\delta/\delta J} + J\right)\int D\phi e^{i\int d^4x\mathcal{L}+J\phi}$$

where $S = \int d^4\mathcal{L}$ is the action. Rewriting

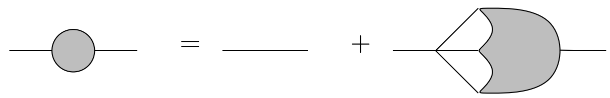
$$JZ[J] = -\frac{\delta S}{\delta\phi}\bigg|_{\phi=\delta/\delta J} Z[J] \tag{1}$$

This is the first version of the Dyson-Schwinger equation. But how do we see the diagrammatic picture that we had in the previous classes? To see this we need to understand $\delta S/\delta\phi$. Let's stick to the example of ϕ^4 theory since that's the only theory for which we've studied the path integral in detail. It turns out that

$$\frac{\delta S}{\delta\phi} = -(\partial^2 + m^2)\phi - \frac{\lambda}{3!}\phi^3$$

the first term of this is essentially by the calculation we did of the propagator (note the error in class where I didn't take the ϕ derivative of the second term!). Note also that the right hand side is the thing that would be zero for the Euler Lagrange equation for ϕ which is the classical equation of motion. Then we can view the Dyson-Schwinger equations as upgrading this equation to all the n-point functions and so as being the quantum equations of motion.

Now to get a diagrammatic version of (1) take ϕ derivatives of each side to get n-point functions, that is to get sums of all Feynman diagrams with n external edges each weighted by their Feynman integral. That is just what the left hand side is, so what about the right hand side? Thinking of the Legendre transform proof we see that the first term is pulling out just a propagator while the second term is pulling out three marked external vertices and gluing them with λ into a new vertex (when lined up with one of the external edges it will be correctly of degree 4). That is we get an equation of the form



Using $Z[J] = e^{W[J]}$ we can get Dyson-Schwinger equations for connected n-point functions and sending it all through the Legendre transform we get Dyson-Schwinger equations for the 1PI case. In both cases we can expand to get diagrammatic equations, the second of which are the diagrammatic equations we were dealing with earlier.

We closed with a particular example that we'll talk about more next time:

$$G(x, L) = 1 - \frac{x}{q^2} \int d^4x \frac{k \cdot q}{k^2 G(x, \log(k^2/\mu^2))(k+q)^2} - \text{same integrand}\Big|_{q^2=\mu^2}$$

where $L = \log(q^2/\mu^2)$.

Next time

Next class we'll talk about the chord diagram solution to the last example.

References

Two sets of notes you can find on the internet are arXiv:1009.4337 by Erik Swanson (see section V.A) and `physik.uni-graz.at/~mqh/notes/DerivationDSEs.pdf` by M.Q. Huber.