# COMBINATORICS OF FEYNMAN DIAGRAMS LECTURE 27 SUMMARY 

WINTER 2018

## Summary

With notation and assumptions from the end of last class, Foissy's theorem is
Theorem 1. The following are equivalent:
(1) $T(x)=x B_{+}(f(T(x))$ is Hopf
(2) $f(z)$ is one of the following

- $f(z)=1$, or
- $f(z)=e^{\alpha z}, \alpha \neq 0$, or
- $f(z)=(1-\alpha \beta z)^{-\frac{1}{\beta}}, \alpha \beta \neq 0$.
(3) There exists $\alpha, \beta$ such that

$$
\begin{aligned}
(1-\alpha \beta z) f^{\prime}(z) & =\alpha f(z) \\
f(0) & =1
\end{aligned}
$$

We went over some of the key ideas in the proof. Some of the calculations are also in your assignment. Then we finished with the binary tree example up to order 4 just checking by hand that it really is Hopf.

Next time
Next class we'll think about more physical DSEs.

## References

Foissy's paper is arXiv:0707.1204

