COMBINATORICS OF FEYNMAN DIAGRAMS LECTURE 25 SUMMARY

WINTER 2018

Summary

We'd just started this new section of the course last class with the definition of graph insertion. We defined the *insertion* of G_1 into G_2, $G_1 \circ G_2$ to be the sum over all possible ways to insert G_1 into G_2, where a way to insert is as follows. If the external leg structure of G_1 is an edge of the theory then you can insert G_1 into any internal edge of that type in G_2 (one way for an oriented edge type and two ways for an unoriented type) and if the external leg structure of G_1 is a vertex of the theory then you can insert G_1 into any vertex of that type in G_2, once for each type-preserving bijection of the external half edges of G_1 with the half edges of the vertex.

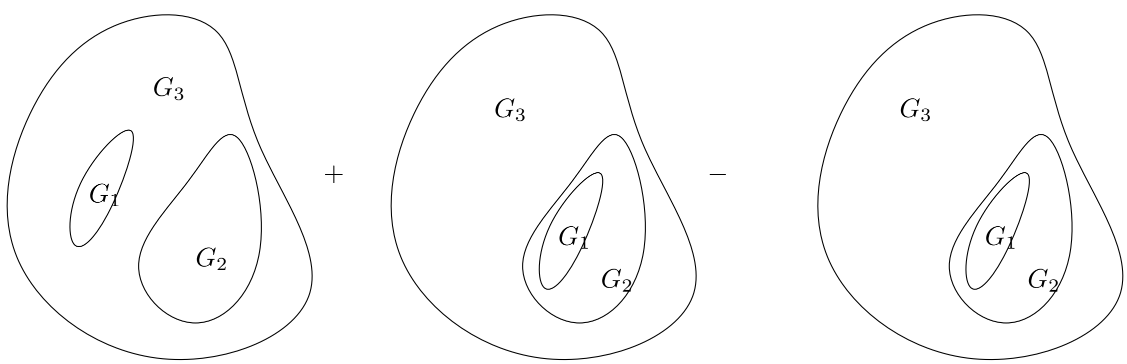
We began today with some examples. Then we proved

Proposition 1.

$$G_1 \circ (G_2 \circ G_3) - (G_1 \circ G_2) \circ G_3 = G_2 \circ (G_1 \circ G_3) - (G_2 \circ G_1) \circ G_3$$

that is, $\circ$ is a pre-Lie product.

Here's a picture that shows you this proof:



This is a picture for the left hand side of the statement. The first two terms are $G_1 \circ (G_2 \circ G_3)$, first we inserted G_2 into G_3 and then we want to insert G_1, it can either go into the G_2 inside the G_3 or not. The third term is $(G_1 \circ G_2) \circ G_3$. Together only the first term is left. But this term is symmetric in G_1 and G_2 and so equals the other side of the statement.

Now thinking of insertion trees, the outermost primitive graph is what everything else is inserted into. So $B_+^\gamma(G)$ ought to be insertion of the graph G into the primitive graph γ. Unfortunately this is not always a 1-cocycle. First of all for insertion into an unoriented edge we generically get two different graphs and sometimes get two copies of the same graph, but for the 1-cocycle property to work we need a linear combination of graphs where the sum of the coefficients is 1, not 2, so divide by 2 for unoriented edge insertions.

Worse things happen with overlapping divergences. If there is only one vertex in our theory and no automorphisms of the external edges of the vertex then it will work if we choose

$$B_+^\gamma(\kappa) = \sum_G \frac{G}{|\mathrm{ue}|\mathrm{maxf}(G)}$$

where the sum runs over G such that G can be obtained by inserting κ into γ, where ue is the number of unoriented edge insertions, and where $\mathrm{maxf}(G)$ is the number of insertion trees giving G. If there are no overlapping subdivergences then $\mathrm{maxf}(G) = 1$. Note that you may get different insertion trees with the same graphs (the example we did in class had this property).

Next time

Next class we'll think about the combinatorial invariant charge and then on Foissy's work.

References

See arXiv:hep-th/0509135 or section 5.4 of the reference linked beside lecture 19.