COMBINATORICS OF FEYNMAN DIAGRAMS LECTURE 20 SUMMARY

WINTER 2018

Summary

Today we looked at renormalization in a toy model.

Let $\mathcal{H}$ be the Connes-Kreimer Hopf algebra. Define

$$F_s(B_+(f)) = \int_0^\infty \frac{F_z(f)}{s+z} dz$$

where s is a parameter with $s > 0$, and extend F_s as an algebra homomorphism, so $F_s(1) = 1$ and $F_s(t_1 t_2 \cdots t_k) = F_s(t_1)F_s(t_2)\cdots F_s(t_k)$. You did some examples and you get some nested integrals that capture the tree structure. However these integrals diverge. We want to think of z and s as being energy-type parameters and so these are UV divergences. Furthermore, we will fix them by looking only at these things relatively rather than absolutely.

So instead of $F_s(\bullet)$ we want $F_s(\bullet) - F_1(\bullet)$. But that's just $\infty - \infty$ so what I actually mean is subtract the integrands and then integrate. This fixes the problem as we try it in Maple and get $-\log(s)$. We are thinking of the target space of F_s as being some space of formal integral expressions. I don't know the correct axioms here, but it will suffice for now that we can add integrands if we're integrating over the same spaces.

We tried $F_s(B_+(\bullet))$ but just subtracting off F_1 doesn't work. Instead we need to use the antipode. Let R be the map that evaluates the integral at $s = 1$. Then define the following

Definition 1. *For t a tree define*

$$S_R^{F_s}(t) = -R(F_s(t)) - \sum_{\substack{C \text{ antichain} \\ C \neq \emptyset, \{root\}}} S_R^{F_s}\left(\prod_{v\in C} t_v\right) R\left(F_s\left(t - \prod_{v\in C} t_v\right)\right)$$

and extend as an algebra homomorphism. $S_R^{F_s}(t)$ *is called the* counterterm *for t.*

The renormalized Feynman rules *are* $F_{ren} = S_R^{F_s} \star F_s$.

Notice how $S_R^{F_s}$ is like the antipode but with R and F_s applied to things. The renormalized Feynman rules are what we want: if we apply them to a tree we end up with an integral that can actually be done and get a function of s as the answer. We used Maple to try it on the example of $B_+(\bullet)$.

The last thing we did was talk about the project. See the project guidelines pdf in the announcements section of the website.

Next time

Next week is spring break. When we return we'll talk about renormalization Hopf algerbras of graphs.

References

You can find an exposition of this toy model in Erik Panzer's masters thesis arXiv:1202.3552.