COMBINATORICS OF FEYNMAN DIAGRAMS LECTURE 13 SUMMARY

WINTER 2018

Summary

Today we completed our very heuristic introduction to the path integral. Last class we'd left things at quantum mechanics and now we need to move to quantum field theory.

The limitation of quantum mechanics is that particles can't be created or destroyed or have interesting interactions and also it can't handle infinitely many particles. So instead of keeping track of some discrete set of particles we instead for each particle type have a field. This use of the word field is like "vector field" or "electromagnetic field", that is at each point in space time it has a value. If it has scalar values it is a scalar field, likewise there are vector and spinnor fields. The particles themselves are disturbances in the field.

So instead of particles running over paths we have changes in the field. This means that $q(t)$ should go to $\phi(t, y)$ where ϕ is the field, t is the time variable and y the space variables. From now on just use x for (t, y). This is ok, but the issue is going from Dq to $D\phi$. The former was very delicate, the latter is just not defined in any rigorous sense. But it's still a very useful heuristic. We get something like

$$\int D\phi e^{i \int d^D x \mathcal{L}}$$

where D is the dimension of spacetime, and $\mathcal{L}$ is the Lagrangian (strictly speaking it is now a Lagrangian density, but still in QFT it is usually just called a Lagrangian).

Now what does $\mathcal{L}$ look like. Let's stick for now to $D = 4$ and a single scalar field $\phi : \mathbb{R}^4 \to \mathbb{R}$. There's a part of $\mathcal{L}$ that corresponds to ϕ freely propagating. This part is written $\frac{1}{2}(\partial\phi)^2$ meaning $\sum_{\mu=1}^{4}(\partial_\mu \phi)^2$ (this is an instance of Einstein summation which is ubiquitous in this world: sum any matching pairs of indices you see). There may be a mass term $-\frac{1}{2}m^2\phi^2$. There may be interaction terms like $-\frac{\lambda_3}{3!}\phi^3$ or $-\frac{\lambda_4}{4!}\phi^4$ etc. There can also be a creation/annihilation term $J\phi$ but this is not usually put in $\mathcal{L}$, it just gets added to the exponent: $\int D\phi e^{i\int d^D x \mathcal{L} + J\phi}$.

Note if $D = 0$ then there's no integral left in the exponent, and ϕ is just a number, so we might as well call it q again and we're back at the graph counting we did before. That's why that approach to graph counting is called graph counting by 0-dimensional field theory.

See the vocab sheet for a helpful summary and words you may want to know.

Next time

Next time we'll derive Feynman rules. Then we'll be able to go directly from graphs to Feynman integrals and not use the path integral unless we want to.

References

I don't know of a perfect book. There are many QFT textbooks and they generally expect you to derive your intuition from quantum mechanics. For those of you coming at it from a different direction you'll probably need to take a few passes at it, first getting a overall sense of what the calculations are supposed to be doing and what the key words and objects are without worrying about details and then come back for another pass (and another, and . . .)

Zee *Quantum field theory in a nutshell* (2003 Princeton).

Peskin and Schroeder *An introduction to quantum field theory* (1995 Westview). The path integral is chapter 9 in Peskin and Schroeder.