COMBINATORICS OF FEYNMAN DIAGRAMS LECTURE 12 SUMMARY – PART 2

WINTER 2018

Summary

In the second half of today's lecture we began the second topic of the course: Feynman diagrams in quantum field theory. The first part of this topic is a very hand-wavy introduction to the path integral.

Say you're doing the double slit experiment. Where does the photon go? The amplitude for the photon to go from the origin O to a particular point P on the screen is the sum of the amplitude to go from O through slit 1 then to P and the amplitude to go from O through slit 2 then to P. This is the superposition of the possibilities.

Why "amplitude"? These are *probability amplitudes* which means they are complex numbers whose norm squared is the probability (or ultimately the probability density). They are useful because they are additive in appropriate situations (which is what superposition is all about). They are called amplitudes from the original wave equation context.

But the superposition principle applies everwhere not just at the slits so for a particle in quantum mechanics you get something like the following:

The amplitude for the particle in state q_{initial} to end in state q_{final} in time T is $\int Dq(t)e^{i\int_0^T dtL(q,dq/dt)}$.

There are many important comments to make here.

- If you have seen this language before (maybe from quantum information) then you probably write "The amplitude for the particle in state q_{initial} to end in state q_{final} in time T" as something like $\langle q_{\text{final}}|e^{-iHT}|q_{\text{initial}}\rangle$ where H is the Hamiltonian. Note that in this context e^{-iHT} is unitary so this is the "multiply a state by a unitary to get the new state" stuff that you always hear about in quantum information.
- The $Dq(t)$ represents an integral over all paths $q(t)$. So this should be an integral over some function space. But functions spaces are typically infinite dimensional, so this is very delicate mathematically and in fact doesn't have a satisfactory rigorous analytic definition in the cases we will care about.
- What is the thing we are integrating over the paths? I.e. what does $e^{i\int_0^T dtL(q,dq/dt)}$ mean? Think of the double slit again. In the classical limit interference shows up because there is a phase difference betwen the wave that took the short path and the wave that took the long path. This would show up in the form $\sum_{\text{paths}} e^{i\cdot\text{phase}}$, so $\int_0^T dtL(q,dq/dt)$ is somehow behaving like a phase. If you know the right bits of analysis, then you might know that one way to try to evaluate an integral with $e^{i\cdot\text{phase}}$ would be by the method of stationary phase and in particular the classical path will be a critical point of the phase as a function of path. You can also say the classical path is the one which minimizes the *action*. The action is $S = \int dtL$ where L is the Lagrangian, and this is what is going on in the path integral above.

- The Hamiltonian and the Lagrangian are two different ways to define the system (the names come from different formulations of classical mechanics). They are a Legendre transform away from each other.

That very roughly describes the intuition of the path integral in quantum mechanics. The next step is to move to quantum field theory.

Next time

Next time we'll give some intuition for the path integral in quantum field theory and then work out how Feynman rules work in a scalar field theory.

References

Any textbook on quantum field theory will explain the path integral, but they typically expect a very different background than what a typical CO student has. One source which is exceedingly handwavy but consequently may make more sense even without the expected background is Zee *Quantum field theory in a nutshell* (2003 Princeton). A more typical quantum field theory text is Peskin and Schroeder *An introduction to quantum field theory* (1995 Westview). The path integral is chapter 9 in Peskin and Schroeder.