COMBINATORICS OF FEYNMAN DIAGRAMS, WINTER 2018, ASSIGNMENT 5

SOLUTIONS

(1) The answer I was looking for here is that you should get a sum over the spanning forests with two parts multiplied by two factors, one factor is the sum of the external momenta in the first tree of the spanning forest, the second factor is the sum of the external momenta in the second tree of the spanning forest. Then because of L^{-1} you'll also have a determinant in the denominator.

Specifically, if we let the internal edge variable be $a_{i,j}$ between vertices i and j (with vertices labelled as the indices of their external momenta, then you get

$$\Bigg(a_{1,3}a_{2,4}(q_1+q_3)(q_2+q_4) + a_{1,2}a_{3,4}(q_1+q_2)(q_3+q_4) + a_{1,4}a_{2,3}(q_1+q_4)(q_2+q_3)$$

$$+(a_{1,2}a_{1,3} + a_{1,2}a_{2,3} + a_{1,3}a_{2,3})(q_1+q_2+q_3)q_4 + (a_{1,2}a_{2,4} + a_{1,2}a_{1,4} + a_{2,4}a_{1,4})(q_1+q_2+q_4)q_3$$

$$+(a_{1,3}a_{3,4} + a_{1,4}a_{3,4} + a_{3,4}a_{1,3})(q_1+q_3+q_4)q_2 + (a_{2,4}a_{2,3} + a_{2,4}a_{3,4} + a_{2,3}a_{3,4})q_1(q_2+q_3+q_4)\Bigg)/\det(L)$$

subject to $q_1+q_2+q_3+q_4=0$ as usual.

(2) Well, Maple isn't actually my favorite CAS, but it does the job:

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    |\^/|     Maple 2017 (X86 64 LINUX)
._|\|   |/|_. Copyright (c) Maplesoft, a division of Waterloo Maple Inc. 2017
 \  MAPLE  /  All rights reserved. Maple is a trademark of
 <____ ____>  Waterloo Maple Inc.
      |       Type ? for help.
> P := a*((d+f)*(b+c) + e*(d+f+b+c)) + (d+b)*(f+c)*e + d*b*(f+c) + f*c*(d+b);
P := a ((d + f) (b + c) + e (d + f + b + c)) + (d + b) (f + c) e + d b (f + c)

     + f c (d + b)

> assume(a>0);
> assume(b>0);
> assume(c>0);
> assume(d>0);
> assume(e>0);
> assume(f>0);
> I1 := int(1/P^2, a=0..infinity);
I1 := 1/((b~ c~ d~ + b~ c~ e~ + b~ c~ f~ + b~ d~ f~ + b~ e~ f~ + c~ d~ e~

     + c~ d~ f~ + d~ e~ f~)

    (b~ d~ + b~ e~ + b~ f~ + c~ d~ + c~ e~ + c~ f~ + d~ e~ + e~ f~))
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```
> I2 := factor(normal(int(I1, e=0..infinity)));
I2 := (ln(d~ + f~) + ln(b~ + c~)

     - ln(b~ c~ d~ + b~ c~ f~ + b~ d~ f~ + c~ d~ f~) + ln(f~ + c~)

                                                               2
     + ln(d~ + b~) - ln(d~ + f~ + b~ + c~))/(b~ f~ - c~ d~)

> I3 := factor(normal(int(I2, b=0..infinity)));
I3 := - (ln(f~ + c~) d~ c~ + f~ c~ ln(f~ + c~) + ln(f~ + c~) d~ f~

                       2
     + ln(f~ + c~) f~  + ln(d~ + f~) d~ c~ + ln(d~ + f~) c~ f~

                                             2
     + d~ f~ ln(d~ + f~) + ln(d~ + f~) f~  - ln(d~ + f~ + c~) c~ f~

                                                       2            2
     - ln(d~ + f~ + c~) d~ f~ - ln(d~ + f~ + c~) f~  - ln(f~) f~

     - ln(c~ d~ + c~ f~ + d~ f~) d~ c~ - ln(c~ d~ + c~ f~ + d~ f~) c~ f~

     - ln(c~ d~ + c~ f~ + d~ f~) d~ f~ + ln(c~) c~ f~ + ln(d~) d~ f~)/(f~

    (f~ + c~) (d~ + f~) d~ c~)
> I5 := int(I3, c=0..infinity);
      /          2                 2
I5 := |ln(d~ + f~)  d~ + ln(d~ + f~)  f~ - d~ ln(f~) ln(d~ + f~)
      \

     - f~ ln(f~) ln(d~ + f~) - d~ ln(d~) ln(d~ + f~) - f~ ln(d~) ln(d~ + f~)

                                                       d~ + f~
     + ln(d~) ln(f~) d~ + f~ ln(d~) ln(f~) - dilog(-------) f~
                                                         f~

             d~ + f~    \
     - dilog(-------) d~|/(d~ (d~ + f~) f~)
               d~       /

> subs(f=1, I5);
/          2                 2
|ln(d~ + 1)  d~ + ln(d~ + 1)  - d~ ln(1) ln(d~ + 1) - ln(1) ln(d~ + 1)
\
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     - d~ ln(d~) ln(d~ + 1) - ln(d~) ln(d~ + 1) + ln(d~) ln(1) d~

                                              d~ + 1    \
     + ln(d~) ln(1) - dilog(d~ + 1) - dilog(------) d~|/(d~ (d~ + 1))
                                                d~      /

> int(subs(f=1, I5), d=0..infinity);
   infinity
  /
 |          /          2                 2
 |          |ln(d~ + 1)  d~ + ln(d~ + 1)  - d~ ln(d~) ln(d~ + 1)
 |          \
/
  0

                                                   d~ + 1    \
     - ln(d~) ln(d~ + 1) - dilog(d~ + 1) - dilog(------) d~|/(d~ (d~ + 1)) dd~
                                                     d~      /

> evalf(int(subs(f=1, I5), d=0..infinity));
memory used=350.5MB, alloc=192.3MB, time=9.31
                                  7.212341419

> evalf(6*Zeta(3));
                                  7.212341418
```

Use `Digits:=30` or some other number if you want more digits.

(3) Answers vary. There were so many typos that you almost found fully disjoint ones from each other.