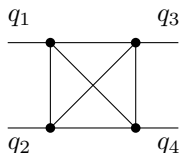


COMBINATORICS OF FEYNMAN DIAGRAMS, WINTER 2018, ASSIGNMENT 5

DUE WEDNESDAY APRIL 4 IN CLASS

There's only one part on this assignment to keep it short since it is the last one and you have your projects to work on. Do all the questions.

- (1) Consider K_4 with external momenta as indicated running left to right:



Let L be the Laplacian with variables and with a matching row and column removed and let B be the vector indexed by vertices of the graph with entry i the external momentum at vertex i with one entry removed (to match the row and column removed from the Laplacian). Calculate $B^T L^{-1} B$ (you can use a computer if you like) and explain how the terms you find relate to spanning forests of the graph with exactly two components.

Clearing denominators by multiplying by the product of all the edge variables, this gives the part of the parametric Feynman integral that we didn't discuss in detail. A general proof of what you observed in this question follows from the all minors matrix tree theorem.

- (2) Use your favorite computer algebra system (CAS) to confirm that the Feynman period of K_4 is $6\zeta(3)$. Write down the denominator of the expression at each step of the integration. Depending on the power of the CAS you use you may need to do the final integral numerically (possibly breaking it into two pieces on either side of a problem point), and then compare numerically to $6\zeta(3)$; this is fine.
- (3) Find at least 5 typos in the lecture summaries or assignment solutions.