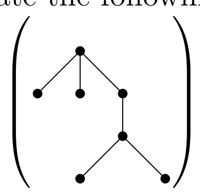
COMBINATORICS OF FEYNMAN DIAGRAMS, WINTER 2018, ASSIGNMENT 3

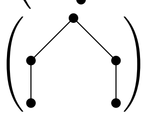
DUE FRIDAY MARCH 2 IN CLASS

PART A

This time you should do all the problems from part A. They are useful for settling your knowledge of the things we've done lately.

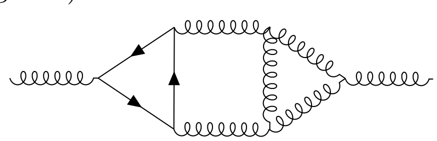
(1) Calculate the following things in the Connes-Kreimer Hopf algebra of rooted trees.

(a) Δ 

(b) S 

(2) Characterize the primitive elements of the Connes-Kreimer Hopf algebra which are homogeneous of degree 3 (note they may be formal sums).

(3) Calculate the coproduct of the following graph in QCD (see the previous assignment for the power counting etc.)



(4) In the tree-based toy model calculate the renormalized Feynman rules applied to the tree $B_+(\bullet\bullet)$. You can use maple or another computer algebra system to actually do the integrals.

PART B

Do all questions from part B.

(1) (a) Let A be an algebra over K. Show that A is commutative if and only if the multiplication of A is a K-algebra homomorphism.
(b) In any bialgebra, if x is a primitive element then $\epsilon(x) = 0$.
(c) In any Hopf algebra, if x is a primitive element then $S(x) = -x$.

(2) The universal property of the Connes-Kreimer Hopf algebra that we discussed in class is due to Connes and Kreimer. A nice exposition of the proof can be found in Theorem 2.4.6 of arXiv:1202.3552. Summarize the proof in a short paragraph (do not give the details).