COMBINATORICS OF FEYNMAN DIAGRAMS, WINTER 2018, ASSIGNMENT 1

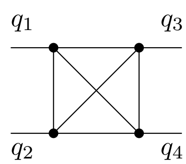
DUE FRIDAY FEB 9 IN CLASS

PART A

Do any two of the three problems from part A.

These problems either concern background things or are routine exercises (possibly both). They will be better exercises for you if you do the ones on the things you *don't* already know, but I leave that choice up to you.

(1) Write down the Feynman integral for



as a Feynman diagram in a scalar field theory with particle mass m.

(2) Let G be a graph. Choose an arbitrary orientation for G and assign a variable p_e for each edge e of G. For each vertex v of G suppose that

$$\sum_{e \sim v} \pm p_e = 0$$

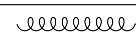
where the sum runs over edges incident to v and the sign is $+$ when the edge goes into v and $-$ otherwise. Prove that any basis of the cycle space of G indexes the free variables of this system of equations.

(3) Consider a scalar field theory with a k-valent vertex, so the edge weight is 2 and the vertex weight is 0. What is the dimension of space-time D as a function of k so that this theory is renormalizable in the combinatorial sense (i.e. the superficial degree of divergence depends only on the external leg structure for connected graphs).

PART B

Do all questions from part B.

(1) The edge and vertex types and power counting weights for quantum chromodynamics (QCD) are

name	graph	weight
gluon		2
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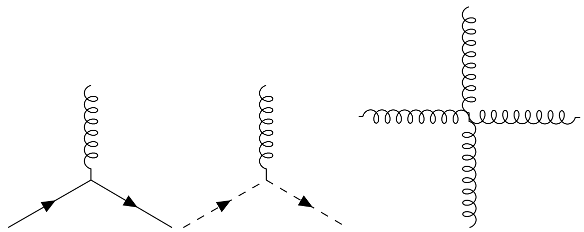
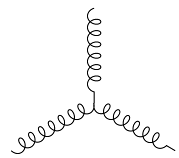
		0
		-1

(a) Prove that QCD is renormalizable in the combinatorial sense in $D = 4$.

(b) List all divergent external leg structures.

(2) How to understand quantum gravity is one of the central concerns of modern fundamental physics. Let's understand in this question why the most naive approach of simply setting up a quantum field theory for gravity doesn't work.

We'll consider just pure gravity (so a graviton field that couples to itself, but no matter fields). As a combinatorial physical theory there is one edge type, an undirected *graviton*, and vertices of any degree are allowed. The edge has power counting weight 2 and the vertex has power counting weight -2.

(a) Prove that pure gravity is not renormalizable in the combinatorial sense in $D = 4$.

(b) Show, in fact, that for every external leg structure there are divergent graphs with that external leg structure.

(c) Give a formula for the superficial degree of divergence of a connected pure gravity graph involving only the dimension of the cycle space of the graph.

(d) Show that the only divergent graphs with no divergent subgraphs are the graphs which are a single cycle potentially with external edges hanging off. (Here the subgraphs you consider should have the following properties. For each vertex v of the subgraph, every half edge of the original graph which is incident to that vertex is also in the subgraph, but two half edges can be in the subgraph without them being joined into an edge even if they were an edge in the original graph. Also, they should be 2-edge-connected.)