CO 330, LECTURE 8 SUMMARY

FALL 2017

Summary

Today we stated and did examples of the Lagrange Implicit Function Theorem (LIFT).

Theorem 1 (Lagrange Implicit Function Theorem)**.** *Let G be a formal power series whose constant term has a multiplicative inverse. Then there is a unique formal power series R satisfying*

$$R(x) = xG(R(x))$$

and for any formal power series F we have

$$[x^n]F(R(x)) = \frac{1}{n}[u^{n-1}]F'(u)G(u)^n \text{ for } n \geq 1$$

and

$$[x^0]F(R(x)) = F(0)$$

A few notes about this. Most of the time we don't really need F, so we'll just take $F(u) = u$. Also typically we'll use this theorem when $R(x)$ is the generating series for some recursively defined class (where the recursive definition is appropriate for obtaining an equation of the form $R(x) = xG(R(x))$ for the generating series.)

With this theorem we can very quickly redo the Catalan calculations we did for binary rooted trees, and we can also do more. We did a couple of examples in class.

The theorem is also very useful in the multivariate case. We used LIFT to calculate the expected number of leaves in a binary rooted tree with n vertices (and to consider what happens as $n \to \infty$). The course notes, in example 8.6, do the analogous calculation for ternary rooted trees. Make sure you understand why you can interchange the partial derivatives and coefficient-of operators.

We'll prove LIFT next time.

References

This material can be found in chapter 8 of the course notes.