CO 330, LECTURE 2 SUMMARY

FALL 2017

Summary

Today we continued the example from last time. We found a different decomposition of the same class of trees based on running down the rightmost path. Suppose a given tree has k vertices in this path, then we can view this tree as a k-tuple of things, each thing being a pair of the kth vertex in the path and the left subtree (possibly empty) of that vertex. To get all trees in the class we need to run over all values of k and so we obtain a bijection

$$\mathcal{T} \to \bigcup_{k\geq 1} (\{\bullet\} \times (\mathcal{T} \cup \{\epsilon\}))^k$$

where all the unions are disjoint. We then converted this to an equation of generating functions. We did it the long way in class, but once you're comfortable with it you can just use the various rules we have to go directly to

$$T(x) = \frac{x(T(x)+1)}{1 - x(T(x)+1)}.$$

After some rearranging we see that both this and the equation we got last class are equivalent to

$$xT(x)^2 + T(x)(2x-1) + x = 0$$

which we can solve using the quadratic formula. Doing so we get

$$T(x) = \frac{1 - 2x \pm (1-4x)^{1/2}}{2x}.$$

We can expand $(1-4x)^{1/2}$ using the generalized binomial theorem (see pages 46, 47, 48 of the course notes) and we see that we need to take the negative root from the quadratic formula and finally obtain that the number of these binary rooted trees of size n is $\frac{1}{n+1}\binom{2n}{n}$, which is the nth Catalan number.

Review all of this so that you are comfortable enough with it that we can do similar things without showing all the steps. Be sure to ask if it isn't making sense.

References

Today's recursive decomposition isn't in the course notes but the calculation at the end is, see pages 46, 47, 48. Also, to the extent that my notation doesn't match with the course notes' it is usually inspired by Flajolet and Sedgewick's "Analytic Combinatorics".