CO 330, LECTURE 22 SUMMARY

FALL 2017

Summary

Today we did stations.
Here are the texts from the four stations:

The EPNT involution. Follow the following steps on each of these partitions:

$$(6,5,4,2),(6,5,3),(5,4,3),(5,4)$$

- Use the lego 1×1 pieces to make the Ferrers diagram for the partition on the base plate. *Swap around who is building so that all the group members get to use the lego!*
- Change the colours as necessary so as to mark the x and o boxes in the involution in the proof of the Euler Pentagonal Number theorem. Make a sensible choice for the boxes with both an x and an o.
- Apply the involution by moving pieces. What partition do you get?

Now consider all partitions of 7 with distinct parts and do the following

- Build all partitions of 7 with distinct parts on the base plate at the same time. Use colours as above.
- Match up the partitions according to the involution. What is left over?

A bijection. The *height* of a tree is the number of edges in the longest path from a leaf to the root.

Work together with your group on the following items using your whiteboard

- Write down all partitions of n for $n \leq 5$.
- Draw all rooted trees of height at most 2 with at most 6 vertices.
- Give a bijection between partitions of n and rooted trees of height at most 2 with $n+1$ vertices.

h-index. One way that granting agencies etc. try to judge academic performance is what is known as the h-index.

Suppose an academic has authored papers $p_1, p_2, \ldots, p_k$ which have $s_1, s_2, \ldots, s_k$ citations respectively. Then consider $s_1, s_2, \ldots, s_k$ as a partition. Call this the publication partition. The edge length of the Durfee square of the publication partition is the *h-index*. A more elementary way to say this is that the h-index is the largest number h such that the academic has at least h papers each with at least h citations.

Discuss the following with your group.

- Is the h-index a reasonable way to judge academic success? What facets of academic performance is it measuring?

- What other partition statistics on the publication partition could you use to evaluate an academic's publishing performance? What advantages and disadvantages would they have relative to the h-index.
- What aspects of academic excellence can partition statistics not measure? In what ways can the partition statistics be gamed?

Thinking of academics, have you considered doing a USRA this summer? They're not just for geniuses.

Partition numbers. The question of a closed form expression for the number of partitions of n was open for a long time.

Read the MAA popular article (4 pages) and the OEIS wiki explanation (1 page) about the Bruinier-Ono formula for the number of partitions. The full paper is also here if you want to take a look but it requires background well beyond this course to understand.

When you are all done reading discuss the following.

- Is this a closed form expression? Is it the expression we sought?
- Why was it so hard to find?
- The original title of the full paper was "An algebraic formula for the partition function". Compare this to the actual title on the version you have. Why do you think it might have changed?

References

If you're interested in a USRA, the C&O department information is here `https://uwaterloo.ca/combinatorics-and-optimization/undergraduates/undergraduate-research` with deadline December 1. Other departments will have similar but different procedures and deadlines. As will other universities (for example `http://www.sfu.ca/math/nserc-usra---vpr.html`, they don't have this year's info up yet, but the deadline is typically in January).

Regarding the Bruinier-Ono formula, the references you were looking at were `https://www.maa.org/press/periodicals/maa-focus/road-to-partition-unveiling-the-fractal-structure-of-partition-numbers` and `http://oeis.org/wiki/Partition_function#Partition_function_finite_formulas` and the full paper is `https://arxiv.org/abs/1104.1182`.