CO 330, LECTURE 20 SUMMARY

FALL 2017

Summary

Today we finished the proof of Euler's pentagonal number theorem. The main part of the proof that we did today was define the sign-reversing involution ϕ.

The basic idea of ϕ is that given a partition you put xs on the last row (we used purple squares in class) and put os on the rightmost antidiagonal (we used blue squares in class) (that is put o on the last box in the first row and continue putting os one step down and one step left until there is no box in that position). Then if it can ϕ takes off the xs and puts them as a new rightmost antidiagonal; otherwise if it can ϕ takes off the os and puts them as a new last row; and otherwise does nothing.

There were a lot of details to check to make sure this works out, and then this ends up proving Euler's pentagonal number theorem because ϕ pairs up elements which have opposite sign in $a_n - b_n$, so only the fixed points of ϕ areleft in $A(x) - B(x)$ (the notation is from last time) and the fixed points occur precisely when $n = h(3h - 1)/2$ or $n = h(3h + 1)/2$ with exactly one in each case.

What should you get out of this proof? The most important thing is how sign-reversing involutions can let us prove this kind of result combinatorially. Why ϕ is useful is more important than the details of ϕs definition.

References

Euler's pentagonal number theorem is the first topic of chapter 10 in the course notes.