CO 330, LECTURE 1 SUMMARY

FALL 2017

Summary

Today we began with an overview of administrative things, all of which can be found in the syllabus which is posted on the website `http://www.math.uwaterloo.ca/~kayeats/teaching/co330.html`. Most important to note is the date of the midterm: October 20 in class. We decided on my office hours which will be Mondays 2:30-3:30 and Tuesdays 1:30-2:30.

Then we began looking at the content of the course. The first week or so will serve to set the foundations while also summarizing the enumerative half of 239/249. The key definitions today were the definition of combinatorial class and of ordinary generating function:

Definition 1. *A* combinatorial class *is a set $\mathcal{C}$ with a* size *or* weight *function $\omega : \mathcal{C} \to \mathbb{Z}_{\geq 0}$ such that for all $n \in \mathbb{Z}_{\geq 0}$ the set $\{c \in \mathcal{C} | \omega(c) = n\}$ is finite.*

Definition 2. *Given a combinatorial class $\mathcal{C}$ the* ordinary generating series *of $\mathcal{C}$, notated $\Phi^{\omega}_{\mathcal{C}}(x)$ or $C(x)$ is*

$$\sum_{c \in \mathcal{C}} x^{\omega(c)}$$

After discussing some things about these definitions and giving some examples, we concluded the class with a more substantial example. We let $\mathcal{T}$ be the combinatorial class of binary rooted trees where each vertex has a left and a right child either or both of which may be empty. We observed that there is a decomposition captured by the following bijection

$$\mathcal{T} \to \{\bullet\} \times (\mathcal{T} \cup \{\epsilon\}) \times (\mathcal{T} \cup \{\epsilon\})$$

which describes taking a binary rooted tree and decomposing it into its root, its left child, and its right child. Then using the properties of generating series that we knew in math 239/249 we converted this decomposition into a functional equation for the generating function:

$$B(x) = x(B(x) + 1)^2.$$

This process is very typical of what will happen in this course.

References

To the extent that generating series were not familiar you should review your 239/249 notes. For those hankering for a more rigorous foundation, look at chapters 1 and 4 of the CO 330 course notes. The binary rooted tree example (with slightly different notation) can be found at the beginning of chapter 6 in the CO 330 course notes.