CO 330, LECTURE 10 SUMMARY

FALL 2017

Summary

We've now moved on to the second section of the course which is about q-analogues. To start, though, we wanted to revisit the usual binomial coefficients.

We first talked about how subsets are counted by binomial coefficients. We talked about the handwavy proof you may have seen in 239/249, and then talked about a bijective proof. More details (in pseudocode form) can be found in the course notes.

Then we talked about how lattice paths starting at $(0, 0)$, using the steps $\rightarrow$ and $\uparrow$ and ending at (a, b) are counted by $\binom{a+b}{a}$.

With these two classes of combinatorial objects which are counted by binomial coefficients in hand, we can try to prove binomial identities by interpreting each side as counting some objects and then prove the identity by proving a bijection between the objects counted on each side. In groups you worked on four examples like this. The first three examples were example 3.2, 3.3 and 3.4 in the notes. Notice that the groups doing the third of these used subsets while the notes used lattice paths; there is more than one way to think about it. The fourth example was

$$\binom{a+1+b}{b} = \sum_{j=0}^{b} \binom{c+j}{j}\binom{a-c+b-j}{b-j}$$

To get an interpretation we'll also need the restriction $a \geq c$ and $a, b, c \in \mathbb{Z}_{\geq 0}$. This is the hardest of the four and I left it with you.

References

The rigorous version of the proof that subsets are counted by binomial coefficients is from chapter 2 of the course notes. The rest of what we did was from chapter 3 of the course notes. The fourth identity is a special case of exercise 6 of chapter 3 of the course notes (the specialness is because the arguments need to be restricted to get a combinatorial proof – in the exercise they intend you to use Proposition 3.6 to get the result for more general arguments. We aren't going to cover Proposition 3.6 in class but it is part of one of the part B problems on the assignment).