CO 330, FALL 2017, ASSIGNMENT 8

DUE WEDNESDAY NOVEMBER 29 AT 4PM VIA CROWDMARK

PART A

Do all problems in part A.

(1) This question is about combinatorial classes $\mathcal{C}$ where $C(x)$ has radius of convergence $\rho > 0$.
   (a) Find an example of such a combinatorial class where $\lim_{x\to\rho^-} C(x) = \infty$. Justify.
   (b) Find an example of such a combinatorial class where $\lim_{x\to\rho^-} C(x)$ is finite. Justify.

(2) Implement a Boltzmann generator for binary strings with no 00 substring. What is the maximum value for x? Find the value of x such that the expected word size is 100. Run the algorithm with this value of x until you get a tree in the range 80 - 120. How many times did you have to run it?

PART B

Do **any two** of the following three problems for part B. If you submit more than two only the first two will be graded.

(1) Implement a Boltzmann generator for set partitions; it's ok if you generate just the shape and not the labelling but it should be uniform on the labelled objects (so shapes with more labellings will be more common by the appropriate amount). Find the value of x such that the expected word size is 100. Run the algorithm with this value of x until you get a tree in the range 80 to 120. How many times did you have to run it? Compare your answer to what happend in question A2.

(2) Read section 6 of `http://algo.inria.fr/flajolet/Publications/DuFlLoSc04.pdf` (actually it is a good idea to read the whole paper). Give a summary of the main point of section 6 as a whole and the main point of each subsection of section 6. The summaries for each section and subsection should be at most a modest sized paragraph each and your whole summary should be less than a page.

(3) (a) For each of the following classes plot the expected size of a Boltzmann model with parameter x as a function of x.
   - plane trees
   - binary strings with all blocks of 0s of even length.
   - set partitions (using the exponential generating function).

   (b) The expected value only tells part of the story – we also want to know whether the distribution is more spread out or more concentrated. The *variance* is one measure of this and in our context for a generating function $A(x)$ is given by

$$V_x = \frac{x^2A''(x) + xA'(x)}{A(x)} - \frac{x^2A'(x)^2}{A(x)^2}.$$

For each of the combinatorial classes from the previous part plot the variance of the Boltzmann model as a function of x.

(c) Which combinatorial class from the first part do you think will work best with a Boltzmann generator? Which will be worst?