CO 330, FALL 2017, ASSIGNMENT 6

DUE WEDNESDAY NOVEMBER 15 AT 4PM VIA CROWDMARK

PART A

Do all problems in part A.

(1) (a) Course notes chapter 10 question 1.
  (b) Course notes chapter 10 question 2.

(2) The course notes and the notes `http://people.math.sfu.ca/~kyeats/teaching/math343/labelled.pdf` (which is close to what we did in class in terms of notation), use different notation for operators/constructions on labelled combinatorial classes. Make a dictionary relating the notation between the two sources. If there are slight discrepancies, give the translation that works the bulk of the time but also clarify how the edge cases differ.

PART B

Do **any three** of the following four problems for part B. If you submit more than three only the first three will be graded.

(1) Course notes chapter 10 question 3 – but it needs correcting. The formula should have a $(-1)^h$ in it (otherwise you're summing a bunch of positive things so you can't get 0). That is, for part a you want to prove

$$\sum_{h=-\infty}^{\infty} (-1)^h p\left(n - \frac{h(3h-1)}{2}\right) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases}$$

(2) *(Compare course notes chapter 11 question 5)* A **derangement** is a permutation σ where there is no i such that $\sigma(i) = i$. Let $\mathcal{P}$ be the combinatorial class of permutations, let $\mathcal{I}$ be the combinatorial class of identity permutations, and let $\mathcal{D}$ be the combinatorial class of derangements, all considered as labelled classes.

  (a) Give a combinatorial decomposition involving $\mathcal{P}$, $\mathcal{I}$ and $\mathcal{D}$.
  (b) Prove that

$$D(x) = \frac{e^{-x}}{1-x}$$

  (here $D(x)$ is the exponential generating function)

  (c) Use the previous part to prove that the number of derangements of n is

$$n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}$$

and hence that the probability that a permutation of n is a derangement approaches

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} = \frac{1}{e}$$

as $n \to \infty$.

(3) Find the exponential generating functions for the following classes of set partitions (justify your answers):
 (a) Set partitions with no singleton parts. *(A singleton is a set with exactly one element.)*
 (b) Set partitions where every part has an odd number of elements.
 (c) Set partitions where there are an odd number of parts.

You may use that

$$\sum_{i=0}^{\infty} \frac{x^{2i+1}}{(2i+1)!} = \sinh(x)$$

the hyperbolic sine function.

(4) Read course notes chapter 11 question 12.
 (a) Give a specification involving the combinatorial class described in this problem as a labelled class.
 (b) Do the question itself.