CO 330, FALL 2017, ASSIGNMENT 3

DUE WEDNESDAY OCTOBER 4 AT 4PM VIA CROWDMARK

PART A

Do all problems in part A.

(1) Let $\mathcal{T}$ be the class of ordered rooted trees with red and blue vertices where every red vertex has an even number of blue children and no red children, and every blue vertex has at most 2 blue children and any number of red children.
  (a) Give a specification for $\mathcal{T}$ (it will probably be a system)
  (b) Using a program/package like combstruct for Maple determine t_{300}.

(2) Give a combinatorial proof (that is, a proof using a bijection between some sets of combinatorial objects) of the fact that
$$\binom{n+m}{k} = \sum_{j=0}^{k} \binom{n}{j}\binom{m}{k-j}$$
for $n, m, k \in \mathbb{Z}_{\geq 0}$ with $k \leq n+m$.

PART B

Do **any two** of the following three problems for part B. If you submit more than two only the first two will be graded.

(1) (a) Define the formal power series $\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$. Prove using the rules of formal power series that $\exp((a+b)x) = \exp(ax)\exp(bx)$.
  (b) Suppose $T(x)$ is a formal power series satisfying $T(x) = x\exp(T(x))$ and let c be any invertible element of the ring we are working over. Show that
$$[x^n]\exp(cT(x)) = \frac{c(c+n)^{n-1}}{n!}$$
  Hint: finally a case where you don't want $F(u) = u$ in the Lagrange Implicit Function Theorem.
  (c) Let a, b and $a+b$ be invertible. Prove *Abel's extension of the binomial theorem*
$$(a+b)(a+b+n)^{n-1} = \sum_{k=0}^{n} \binom{n}{k} a(a+k)^{k-1} b(b+n-k)^{n-k-1}$$
  by calculating the coefficient of x^n in $\exp((a+b)T(x))$ in two different ways.

(2) Course notes chapter 3 question 9.

(3) Read proposition 3.6 of the course notes and its proof.
  (a) Do course notes chapter 3 question 5
  (b) Suppose you have a friend in this class who does not understand the difference between part a of this question and question A2. Write a paragraph or two explaining the difference and the idea of proposition 3.6.