CO 330, FALL 2017, ASSIGNMENT 2

DUE WEDNESDAY SEPTEMBER 27 AT 4PM VIA CROWDMARK

PART A

Do all problems in part A.

(1) Let $\mathcal{D}$ be the class of all plane rooted trees where each vertex has either 0 children or at least 2 children.
 (a) Give a combinatorial specification for $\mathcal{D}$, that is, give an equation involving $\mathcal{D}$ using operators like $\times, +, \textsc{Seq}$, and classes $\mathcal{E}, \mathcal{Z}$.
 (b) Give a polynomial equation involving $D(x)$.

(2) Show that the number of trees from part A question 1 with n vertices is

$$\frac{1}{n}\sum_{j=1}^{\lfloor (n-1)/2 \rfloor} \binom{n}{j}\binom{n-2-j}{n-1-2j}$$

PART B

Do **any two** of the following three problems for part B. If you submit more than two only the first two will be graded.

(1) Course notes chapter 8 question 4. First give a specification for the class of trees in the question which is suited to answering the question. (Plane planted trees are what we've been calling ordered rooted trees).

(2) Consider rooted trees where each vertex has k ordered children any of which may be empty, so in particular there are k such trees on two vertices (analogous to the binary rooted trees we saw in class). Prove that as $n \to \infty$ the expected proportion of vertices which are leaves in a random such tree of size n is asymptotic to

$$\frac{(k-1)^k}{k^k}$$

(3) For $n \geq 1$ a $2 \times n$ Standard Young Tableau is a $2 \times n$ matrix where each number $1, 2, \ldots, 2n$ occurs exactly once and where the rows increase from left to right and the columns increase from top to bottom. Show that for $n \geq 1$, the number of $2 \times n$ Standard Young Tableau is $\frac{1}{n+1}\binom{2n}{n}$ by finding a bijection with some other class with these sizes.