CO 330, FALL 2017, ASSIGNMENT 1

DUE WEDNESDAY SEPTEMBER 20 AT 4PM VIA CROWDMARK

The crowdmark links to submit this assignment will be send out in the second week to give enrolment a chance to settle. If you have not received yours by Sept 15 please let me know.

PART A

Do all problems in part A.

(1) (a) Give two different combinatorial classes with an infinite number of elements and the same underlying set.
    (b) Give a third weight function on the same set so that the result is not a combinatorial class

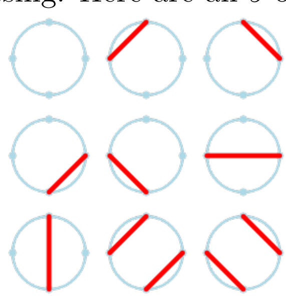
(2) Course notes chapter 4, question 6. Note that in this question the weight function ω maps from S to $\mathbb{N}^2$ and so given $s \in S$ we write $\omega(s) = (\omega_1(s), \omega_2(s))$. Often you just think of ω_1 and ω_2 as two functions on the set S but they don't need to separately satisfy the finiteness condition of a weight function; only ω as a whole needs to satisfy the finiteness condition.

PART B

Do **any three** of the following four problems for part B. If you submit more than three only the first three will be graded.

(1) We have several tools at our disposal to help understand and play with counting sequences. This exercise is to help you gain familiarity with some of these tools.

Let $\mathcal{M}$ be the following class of combinatorial objects: An object of size n is a circle on n points, with some collection chords between the vertices under the condition that no two chords are crossing. Here are all 9 objects of size 4:



(a) Prove that $\mathcal{M}$ satisfies the definition of combinatorial class.
(b) Determine by hand the first five elements of the counting sequence $(M_n)_{n\geq 0}$;
(c) The generating function of this sequence is:

$$M(z) = \frac{1 - z - \sqrt{1 - 2z - 3z^2}}{2z^2}.$$

Using some sort of computer algebra program, determine the first 10 elements of the counting sequence. Some examples: You can use the command `series` or `taylor` in Maple, or the command series in the prompt at the website `http://www.wolframalpha.com/`.

(d) Look up this sequence on the On-line encyclopedia of integer sequences (`http://oeis.org`). What is its sequence number? List two other combinatorial classes also counted by these numbers. Choose one, and draw all the elements of size less than or equal to 5.

(2) Let $\mathcal{B}$ be the class of binary rooted trees where every node has either 0 or 2 children.

(a) Find an equation which $B(x)$ satisfies.

(b) Determine $[x^n]B(x)$

(c) How does this compare to the class of all binary rooted trees from lecture?

(3) Course notes chapter 4, question 5.

(4) The multiplicative inverse of a formal power series $A(x)$ is a formal power series $C(x)$ such that $A(x)C(x) = 1$.

(a) Prove that a formal power series $A(x) = \sum_{n=0}^{\infty} a_n x^n$ with coefficients in a commutative ring has a multiplicative inverse if and only if a_0 is invertible in the ring. Furthermore, when it exists the inverse is unique.

(b) Give a rigorous meaning to the expression $1 + A(x) + A(x)^2 + A(x)^3 + \cdots$ by defining a valid operation on formal power series which can be viewed as taking $A(x)$ to $1 + A(x) + A(x)^2 + A(x)^3 + \cdots$.

(c) Let $A(x)$ be a formal power series with zero constant term. Prove that the multiplicative invese of $1 - A(x)$ is $1 + A(x) + A(x)^2 + A(x)^3 + \cdots$. As a consequence it makes sense to say that

$$\frac{1}{1 - A(x)} = 1 + A(x) + A(x)^2 + A(x)^3 + \cdots$$

which is our Sequence operator.