CO749 **Assignment 2** **Due: October 19**

Problem 1: Prove that the following are equivalent:

(a) Each minor-closed class of graphs has only finitely many excluded minors (up to isomorphism).

(b) There are only countably many minor-closed classes of graphs.

(c) For any minor-closed class of graphs the membership testing problem is decidable.

Problem 2: Let $\mathcal{T}$ be a tangle in a graph G, let $(G_1, G_2) \in \mathcal{T}$, and let $X = V(G_1 \cap G_2)$. Prove that, if $V(G_1)$ is $\mathcal{T}$-closed and $|X| = \kappa_{\mathcal{T}}(V(G_1))$, then X is a highly connected set in G_2.

Problem 3: Let X and Y be vertex sets in a graph G and let $P_1, \ldots, P_k$ be disjoint (X, Y)-paths in G where $P_1 = (v_0, v_1, \ldots, v_t)$. Show that, if $\kappa_{G-v_i}(X, Y) < k$ for each $i \in \{1, \ldots, t-1\}$, then there is a k-dissection $(H_1, \ldots, H_t)$ such that

(i) $X \subseteq V(H_1)$ and $Y \subseteq V(H_t)$, and

(ii) $v_i \in V(H_i \cap H_{i+1})$ for each $i \in \{1, \ldots, t-1\}$.

Problem 4: Let $t, n \in \mathbf{Z}_+$ and let $k = k(t, n)$, where $k(t, n)$ is a function of your choosing. Let $S_1, \ldots, S_t, T_1, \ldots, T_t$ be sets of vertices in a graph G such that $\kappa_G(S_i, T_i) \geq k$ for each $i \in \{i, \ldots, t\}$. Show that either

(i) there are disjoint paths $P_1, \ldots, P_t$ where P_i is an (S_i, T_i)-path for each $i \in \{1, \ldots, t\}$, or

(ii) G has an $n \times n$-grid-minor.

Problem 5: Let $n \in \mathbf{Z}_+$ and let $k = k(n)$ be a function of your choosing. Let $\mathcal{T}$ be a tangle in G and let H be a minor of G that is isomorphic to a square grid. Show that, if $\kappa_{\mathcal{T}}(V(H)) \geq k$, then G has a minor H' isomorphic to the $n \times n$-grid such that $V(H')$ is $\mathcal{T}$-independent.