C&O 769, Topics in Semidefinite Programming and Applications (Winter 2013)
Assignment 2
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Due: Tueday Mar. 12, 1:00PM (before class),

1 Linear Transformations

1. Consider the linear transformation

$$\mathcal{A} : \mathbb{R}^{n^2} \to \mathbb{R}^{n^2}, \qquad \mathcal{A}(x) = \mathrm{vec}(U\,\mathrm{Mat}(x)U),$$

   where $U \in \mathcal{S}^n$, $\mathrm{vec}(X)$ is the vectorization of the $n \times n$ matrix X, columnwise, and $\mathrm{Mat} = \mathrm{vec}^{-1}$ forms the matrix from the column vector.

   (a) Find the *adjoint* of $\mathcal{A}$. (Deduce that $\mathcal{A}$ is self-adjoint.)

   (b) Form the matrix representation of $\mathcal{A}$ explicitly in terms of U. (Hint: Use $\otimes$. And note that $\mathcal{A}$ is symmetric.)

   (c) Suppose that $U \succeq 0$. Show that $\mathcal{A}$ is a positive semidefinite operator, i.e., show that $\langle x, \mathcal{A}x \rangle \geq 0, \forall x \in \mathbb{R}^{n^2}$.

2. Consider the quadratic form $q : \mathbb{M}^n \to \mathbb{R}$

$$q(X) = \langle X, AXB \rangle,$$

   where $X \in \mathbb{M}^n$ the space of $n \times n$ real matrices and $A, B \in \mathcal{S}^n$. Can you extend the above statements to this quadratic form?

2 Nearest Matrix Problems and SNL

Consider the SNL problem with n sensors and m anchors. Assume that you have partial information based on a given radio range.

1. Write down the SNL problem as an ℓ_2 nearest matrix completion problem. (Use the Gram matrix $B = PP^T$ and the EDM representation $D = \mathcal{K}(B)$ to obtain a quadratic objective function.)

2. Generate some random data with $n = 20, m = 5$. Solve the SDP relaxation using CVX. Use the optimal solution to get an estimate for the SNL optimum.