(9.12) Notice that in this question, $\nabla^2 f$ (or $H$) is not necessarily singular. Though most of you point out that $\beta$ is the Lagrangian multiplier, to give a strict proof, you should consider the following different cases: $H$ is singular or $H$ is invertible; the norm of the unconstrained minimizer is greater than or not greater than $\gamma$.

(11.2) The associated centering problem

$$\min tx_2 - \log(x_2 - x_1) - \log x_2$$

is actually unbounded below as $x_1 \to -\infty$.

(11.3) To solve this question, you may first assume that the sublevel set of the centering problem is unbounded. Then a point $x$ and a direction $v$ can be found in this set such that $x + sv$ are in the sublevel set for $\forall s \geq 0$. Since the sublevel set of the original problem is bounded, $f_0(x + sv)$ is increasing for $s$ sufficiently large. Therefore, we may choose $x$ such that $\nabla f(x)^T v > 0$. Contradiction then can be found by some simple calculation.

(11.4) Because $x^T x \leq R^2$, 

$$\frac{1}{R^2 - x^T x} I \succeq \frac{1}{R^2} I$$

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