Quadratic program (QP)

minimize
$$(1/2)x^TPx + q^Tx + r$$

subject to $Gx \leq h$
 $Ax = b$

- $P \in \mathbf{S}_{+}^{n}$, so objective is convex quadratic
- minimize a convex quadratic function over a polyhedron



Examples

least-squares

minimize $||Ax - b||_2^2$

- analytical solution $x^* = A^{\dagger}b$ (A^{\dagger} is pseudo-inverse)
- can add linear constraints, e.g., $l \preceq x \preceq u$

linear program with random cost

minimize
$$\bar{c}^T x + \gamma x^T \Sigma x = \mathbf{E} c^T x + \gamma \operatorname{var}(c^T x)$$

subject to $Gx \leq h$, $Ax = b$

- c is random vector with mean \bar{c} and covariance Σ
- hence, $c^T x$ is random variable with mean $\bar{c}^T x$ and variance $x^T \Sigma x$
- $\gamma > 0$ is risk aversion parameter; controls the trade-off between expected cost and variance (risk)