Block Gaussian Elimination ... but BEWARE of typos!!!

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Outline

1. Block Elimination

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Elementary Matrices

Given blocked matrix B

$$B := \begin{bmatrix} H & A^T \\ A & 0 \end{bmatrix}, \qquad H \succ 0, A \text{ full row rank}$$

start block Gaussian elimination

$$E_1 := \begin{bmatrix} H^{-1} & 0 \\ 0 & 0 \end{bmatrix},$$

$$E_1 B = \begin{bmatrix} H^{-1} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} H & A^T \\ A & 0 \end{bmatrix} = \begin{bmatrix} I & H^{-1}A^T \\ A & 0 \end{bmatrix},$$

Block GE cont...

$$E_2 := \begin{bmatrix} I & 0 \\ -A & I \end{bmatrix}$$

$$E_2(E_1 B) = \begin{bmatrix} I & 0 \\ -A & I \end{bmatrix} \begin{bmatrix} I & H^{-1}A^T \\ A & 0 \end{bmatrix} = \begin{bmatrix} I & H^{-1}A^T \\ 0 & -AH^{-1}A^T \end{bmatrix}$$

Block GE cont...

$$E_3 := \begin{bmatrix} I & H^{-1}A^T(AH^{-1}A^T)^{-1} \\ -A & -(AH^{-1}A^T)^{-1} \end{bmatrix}$$

$$E_3(E_2 E_1 B) = E_3 \begin{bmatrix} I & H^{-1}A^T \\ 0 & -AH^{-1}A^T \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

Inverse is $E_3 E_2 E_1$

Final Inverse

Inverse is $E_3 E_2 E_1$

$$\begin{bmatrix} I & H^{-1}A^T(AH^{-1}A^T)^{-1} \\ -A & -(AH^{-1}A^T)^{-1} \end{bmatrix} \begin{bmatrix} I & 0 \\ -A & I \end{bmatrix} \begin{bmatrix} H^{-1} & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} I & H^{-1}A^T(AH^{-1}A^T)^{-1} \\ -A & -(AH^{-1}A^T)^{-1} \end{bmatrix} \begin{bmatrix} H^{-1} & 0 \\ -AH^{-1} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} H^{-1} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} H^{-1}A^T \\ -I \end{bmatrix} \left[AH^{-1}A^T \right]^{-1} \begin{bmatrix} H^{-1}A^T \\ -I \end{bmatrix}^T$$

Thanks for your attention!

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