# CO 602/CM 740: Fundamentals of Optimization
# Problem Set 4

H. Wolkowicz

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Due: Thursday 2016-Oct-27 before 10AM lecture starts.

## Contents



## 1 Unique Optimum

Consider a linear programming problem in standard and suppose that $x^*$ is an optimal basic feasible solution. Consider an optimal basis associated with $x^*$. Let $B$ and $N$ be the set of basic and nonbasic indices, respectively. Let $I$ be the set of nonbasic indices $i$ for which the corresponding reduced costs are zero.

1. Show that if $I$ is empty, then $x^*$ is the only optimal solution.

2. Show that $x^*$ is the unique optimal solution if, and only if, the following linear problem has an optimal value of zero:

$$\begin{array}{lll} \max & \sum_{i\in I} x_i & \\ \text{s.t.} & Ax = b & \\ & x_i = 0, & \forall i \in N\backslash I, \\ & x_i \geq 0, & \forall i \in B \cup I. \end{array}$$

## 2 Transportation Problem; Economic Interpretation

1. Model the transportation problem of shipping new cars with supplies at each warehouses being $s_i, i = 1, \ldots, m$, demand at each dealer being $d_j, j = 1, \ldots, n$, and transportation costs given in the matrix elements $C_{ij}, \forall ij$. (Take into account the case when the problem is *not balanced, i.e., when sum of supplies and demands are not necessarily equal.*)

2. Consider the cost, supply, demand data

$$C = \begin{pmatrix} 5 & 6 & 10 & 7 & 6 & 11 & 5 & 4 \\ 9 & 8 & 7 & 7 & 10 & 8 & 7 & 4 \\ 4 & 10 & 6 & 10 & 7 & 8 & 4 & 4 \\ 9 & 3 & 6 & 10 & 6 & 8 & 10 & 6 \\ 4 & 11 & 5 & 8 & 11 & 4 & 4 & 5 \end{pmatrix}, \qquad s = \begin{pmatrix} 9 \\ 4 \\ 2 \\ 9 \\ 10 \end{pmatrix}, \qquad d = \begin{pmatrix} 4 \\ 1 \\ 3 \\ 4 \\ 6 \\ 3 \\ 6 \\ 7 \end{pmatrix}$$

Use MATLAB and solve the problem. Can you ensure that you get integer solutions in MATLAB? Explain how and why.

3. Find the dual optimal solution for the problem in part 2 above. Provide an economic interpretation.

# 3 Degeneracy

Consider the maximization LP

$$\begin{array}{ll} \max & 2.3x_1 + 2.15x_2 - 13.55x_3 - 0.4x_4 \\ \text{s.t.} & \begin{bmatrix} 0.4 & 0.2 & -1.4 & -0.2 \\ -7.8 & -1.4 & 7.8 & 0.4 \end{bmatrix} x \le \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ & x \ge 0 \end{array}$$

1. Show that the simplex method *cycles* for this LP, i.e., it does *not* terminate in a finite number of iterations.[1]

2. can you *perturb* the right hand side by a small amount and stop in a finite number of steps?

3. Do you get a solution (does the algorithm stop) using linprog in MATLAB? (Show your output.)

[1]Hint: Let $x_1$ enter the basis for the first iteration; and, let $x_2$ enter the basis for the second iteration; break the tie by choosing the larger pivot element, i.e., the usual choice for stability of pivots in Gaussian elimination.