CO 602/CM 740: Fundamentals of Optimization
23 minute Quiz,

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Consistent Rounding

Consider the matrix

$$B = \begin{bmatrix} 3.1 & 6.8 \\ 9.6 & 2.5 \end{bmatrix}$$

Let $Be = \alpha, B^T e = \beta$ denote the row, column sums of B, respectively.

1. **(3 Marks)** Use a network flow approach and formulate a mathematical model for the problem of consistent rounding of B, i.e. the problem is to round the elements of B (up or down) in order to obtain the rounded (up or down) row and column sums of B,

   $$\text{round}(B)e = \text{round}(Be), \qquad \text{round}(B^T)e = \text{round}(B^T e),$$

   where round(v) refers to the rounding process on the elements of the vector or matrix v. Write down both a mathematical model, call it (P), and the corresponding directed graph.

2. **(3 Marks)** Transform the model for Item 1 (if needed) to formulate the mathematical model as a *max-flow* or *network flow* problem. (It might help to consider rounding up being preferable to rounding down.)

3. **(4 Marks)** Suppose that A is a general $m \times n$ matrix with rational elements. Prove that the model (P) for consistent rounding from Item 1, above, always has a feasible solution; or, provide a counterexample.

4. **(BONUS)** Use the F-F algorithm or network simplex method with phase I to solve problem (P) with the given matrix B.