CO 602/CM 740: Fundamentals of Optimization
Problem Set 5

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Due: Mon, 2011-Oct-31, by midnight.

Contents

1 Dual LP Optimum from Primal Optimum

Consider the primal problem $\min c^T x$ s.t. $Ax \geq b, x \geq 0$ and suppose that we are given an $\bar{x}$ that is an optimal nondegenerate BFS solution. Write a computer program to find an optimal solution for the dual by solving a linear system of equations. Your program should confirm primal and dual nondegeneracy (strict complementarity) and optimality. Use your program to solve the three problems in the mat files finddualprobi.mat, i=1,2,3, at the website.

Can you suggest a way to do this for a possibly degenerate optimal solution?

2 Hyperplane Separation, Ex 4.34 in Text

Consider a polyhedron P that has at least one extreme point. Suppose that $P = \{x \in \mathbb{R}^n : a_i^T x \geq b_i, i = 1, \ldots, m\}$. Suppose that $0 \notin P$. Explain how a separating hyperplane can be found.

3 The Caterer Problem, Ex 7.1 in Text

A catering company must provide to a client r_i tablecloths on each of N consecutive days. The catering company can buy new tablecloths at a price of p

dollars each, or launder the used ones. Laundering can be done at a fast service facility that makes the tablecloths unavailable for the next n days and costs f dollars per tablecloth, or at a slower facility that makes tablecloths unavailable for the next m days (with $m > n$) at a cost of g dollars per tablecloth ($g < f$). The caterer's problem is to decide how to meet the client's demand at minimum cost, starting with no tablecloths and under the assumption that any leftover tablecloths have no value.

1. Show that the problem can be formulated as a network flow problem. (Hint: Use a node corresponding to clean tablecloths and a node corresponding to dirty tablecloths for each day; more notes may also be needed.

2. Show explicitly the form of the network if $N = 5, n = 1, m = 3$.

4 Problems to Consider; Do not hand in

Two important theorems for LP are connected to complementary slackness and boundedness of the feasible sets. Text: 4.20 and 4.21.