CO 602/CM 740: Fundamentals of Optimization
Problem Set 3

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Contents

1 Optimality Conditions for LP

1. Suppose that the simplex method finds a BFS x such that exactly one reduced cost is negative and all the other reduced costs are nonnegative. Under what conditions can this BFS be optimal. Under what conditions is it definitely not optimal?

2. Suppose that $\bar{x}$ solves an LP in standard form. Let B, N denote the basic and nonbasic indices corresponding to an optimal basis for $\bar{x}$. Let $\mathcal{Z} \subset N$ denote the nonbasic indices for which the reduced costs $\bar{c}_i = 0$.

   (a) Prove that $\mathcal{Z} = \emptyset$ implies $\bar{x}$ is the unique optimum.

   (b) Let e be the vector of ones of appropriate length. Consider the LP

$$\begin{array}{rll} p^* := & \max & e^T x_{\mathcal{Z}} \\ & \text{s.t.} & Ax = b \\ & & x_i = 0, \quad \forall i \in N \backslash \mathcal{Z} \\ & & x_i \geq 0, \quad \forall i \in B \cup \mathcal{Z} \end{array} \tag{1}$$

   Show that $\bar{x}$ is the unique optimum of the LP if, and only if, $p^* = 0$.

2 Solving an LP

Consider the LP

$$\begin{array}{ll} \max & 2y_1 + 4y_2 + y_3 + y_4 \\ \text{s.t.} & \begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 \end{bmatrix} y \leq \begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix} \\ & y \geq 0 \end{array} \qquad (2)$$

1. Solve the LP in (2) using the revised simplex method (cf Section 3.3, page 95 of the text). Show your work and compare your solution *carefully* with that obtained from using linprog in MATLAB (try help linprog); and also compare it with that obtained with the pivoting help from the MATLAB file
orion.math.uwaterloo.ca/˜hwolkowi/henry/teaching/f11/602.f11/pivotLP.m

2. What are the maximum changes that can be made to the elements of $b = (4, 3, 3)^T$ while maintaining the current optimal basis. How does the objective value change for small changes in b?

3. Repeat the questions in Item 2 with b changed to the elements in the objective function $c = (2, 4, 1, 1)$.

3 Degeneracy

Consider the LP

$$\begin{array}{ll} \max & 2.3y_1 + 2.15y_2 - 13.55y_3 - 0.4y_4 \\ \text{s.t.} & \begin{bmatrix} 0.4 & 0.2 & -1.4 & -0.2 \\ -7.8 & -1.4 & 7.8 & 0.4 \end{bmatrix} y \leq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ & y \geq 0 \end{array} \qquad (3)$$

Show that the simplex method *cycles* for this LP, i.e. does *not* terminate in a finite number of iterations. (Hint: Let y_1 enter the basis for the first iteration; and, let y_2 enter the basis for the second iteration. In case of a tie for the choice of leaving variable for the second iteration, break the tie by choosing the larger pivot element, i.e. the usual choice for stability of pivots.)