C&O 463/663 Convex Optimization and Analysis (Fall 2009)

Assignment 1 - Additional Problems

Instructor: Dr. Henry Wolkowicz
Due Date: Tuesday, Sept. 29, 2009

1 Convex Functions and Convex Sets

Recall that $\operatorname{dom} f = \{x : f(x) < +\infty\}$.

1. Suppose that $f : \mathbb{R}^n \rightarrow \mathbb{R} \cup +\infty$ is convex with $\operatorname{dom} f = \Omega \subset \mathbb{R}^n$, an open set. Show that f is continuous on Ω.

2. Suppose that $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is convex, ($\operatorname{dom} f = \mathbb{R}^n$) and bounded above on $\mathbb{R}^n$. Show that f is constant.

3. Suppose that $f : \mathbb{R} \rightarrow \mathbb{R} \cup +\infty$ is convex with $\mathbb{R}_+ \subseteq \operatorname{dom} f$. Show that the running average F is convex, where

$$F(x) := \frac{1}{x} \int_0^x f(t)dt, \quad \operatorname{dom} F = \mathbb{R}_{++}$$

4. What is the distance between two parallel hyperplanes?

2 Convex Optimization Problems

1. Consider the *Best Uniform Approximation Problem* (Tschebyshev approximation): given an $m \times n$ matrix A, and a vector $b \in \mathbb{R}^m$,

$$\min_x \|Ax - b\|_\infty,$$

   i.e. the norm is the infinity norm. This objective function is nonlinear. Rephrase this problem as an LP problem.

2. (*) Suppose a matrix $A \in \mathcal{S}^n_+$ satisfies $I \succeq A$. Prove that the iteration

$$Y_0 = 0, Y_{n+1} = \frac{1}{2}(A + Y_n^2) \qquad (n = 0, 1, 2, \ldots)$$

   is nondecreasing (that is $Y_{n+1} \succeq Y_n$ for all n) and converges to the matrix $I - (I - A)^{1/2}$. (Hint. Consider diagonal matrices A.)