CO350 Linear Programming Chapter 4: Introduction to Duality

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The Dual Problem

Instead of solving an LP problem, we consider a seemingly easier task.

> To decide how close a given feasible solution is to being optimal.

<u>Example: LP in SEF (pg 35)</u>

$$
(P)\quad
\begin{array}{lrcrcrcrcrcl}
\text{maximize} & 5x_1 & - & 6x_2 & + & 8x_3 & + & 4x_4 & + & x_5 & & \\
\text{subject to} & 2x_1 & - & x_2 & + & x_3 & + & x_4 & & & = & 1 \\
 & x_1 & + & 3x_2 & - & x_3 & + & x_4 & + & x_5 & = & 9 \\
 & 2x_1 & & & + & 3x_3 & + & x_4 & + & x_5 & = & 6 \\
 & x_1 & , & x_2 & , & x_3 & , & x_4 & , & x_5 & \geq & 0
\end{array}
$$

Given feasible solution $x^* = [0, 1, 0, 2, 4]^T$. (Check feasible!)
It has objective value 6. (Implication: Optimal value ≥ 6.)

Is x^* optimal? Is x^* close to being optimal?

If we know optimal value ≤ 6, then x^* is optimal.
If we know optimal value ≤ 7, then x^* is near optimal.

Goal: Get a good upper bound on the objective function

$$5x_1 - 6x_2 + 8x_3 + 4x_4 + x_5$$

subject to all LP constraints.